An Interactive Learning System

Algebra Tiles PLAYGROUND

Written in California by Fred Ventura, Ph.D.

This manual provides additional support materials for educators who are using the Algebra Tiles Playground App for iPad.

Ventura Educational Systems

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Some of our apps keep records of individual student scores while the app is being used. By tapping a scoreboard icon the student, teacher or parent can view the performance record. This data can be erased by tapping an eraser icon. The purpose for the scoreboard screen is to :

- Help assess the student's strengths and weaknesses.
- Help the user decide to change/correct the instructional level selected in the app.
- Help students self-evaluate their instructional needs.
- Help teachers and parents better plan lessons.

Security

The scores that students get when taking the quizzes that are a part of our apps are only stored locally. Anyone who has access to the device can see the information unless it has been erased. Once the records are erased by the user, they are no longer accessible.

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A Note to Teachers…

The secondary mathematics curriculum traditionally has been designed to provide students with the opportunity to develop skills and acquire knowledge that would be important in adulthood and to prepare students to enter careers that would not require a high level of mathematical ability. Today the context of mathematics instruction needs to be much broader. The accelerated pace at which modern society produces technological change requires that high school graduates be prepared for careers in which mathematics will be very important. In order to be successful in many careers students will need to be confident in their mathematical abilities. They will need to be able to solve problems, to communicate mathematical ideas and to think logically.

The National Council of Teachers of Mathematics (NCTM) in the Curriculum and Evaluation Standards for School Mathematics calls for the establishment of a framework for a core curriculum in grades 9-12 that reflects the needs of all students. The second standard of the NCTM document address the need for the mathematics curriculum to…

Include the continued development of the language and symbolism to communicate mathematical ideas so that all students can —

- ❑ Reflect upon and clarify their thinking about mathematical ideas and relationships;
- ❑ Formulate mathematical definitions and express generalizations discovered through investigations;
- ❑ Express mathematical ideas orally and in writing;
- ❑ Read written presentations of mathematics with understanding;
- ❑ Ask clarifying and extending questions related to mathematics the have read or heard about;

  Appreciate the economy, power and elegance of mathematical notation and its role in the development of mathematical ideas.

(NCTM)

Algebra Tiles Playground is a iOS App that is intended to help students develop their ability to understand algebra na communicate algebraic concepts. It is specifically designed to create a learning environment where the vocabulary of algebra can be learned in an enjoyable way. Algebra Tiles give students the ability to use a computer as a tool for exploring algebraic ideas. This type of open-ended interactive app is most effective if the students work in groups of two so that the discussion of mathematical ideas is facilitated. Comments and feedback on the effectiveness of this app and supporting material are always welcome.

The Algebra Tiles Playground is an interactive learning system that has been designed to provide 7th grade enrichment throughout adult level instruction in mathematics. Several approaches to algebra instruction are combined in the design of this educational app. Algebra Tiles Playground is designed to make a bridge to abstract mathematical reasoning with Algebra Tiles. The Algebra Tiles Playground is a great way for algebra teachers to introduce the abstract concepts related to operations with binomials and trinomials. An actual physical set of algebra tiles usually includes squares and rectangles of different sizes. In the Algebra Tiles Playground black tiles are positive and red tiles are negative. The small square is a positive or negative unit. The rectangle represents the x-term. It can be positive or negative. The large square represents x^2. It can also be positive or negative.

The tiles are used to represent expressions and also show operations with binomials and trinomials. See below for some teaching ideas regarding how to use the Algebra Tiles Playground to help students better understand some basic concepts of algebra.

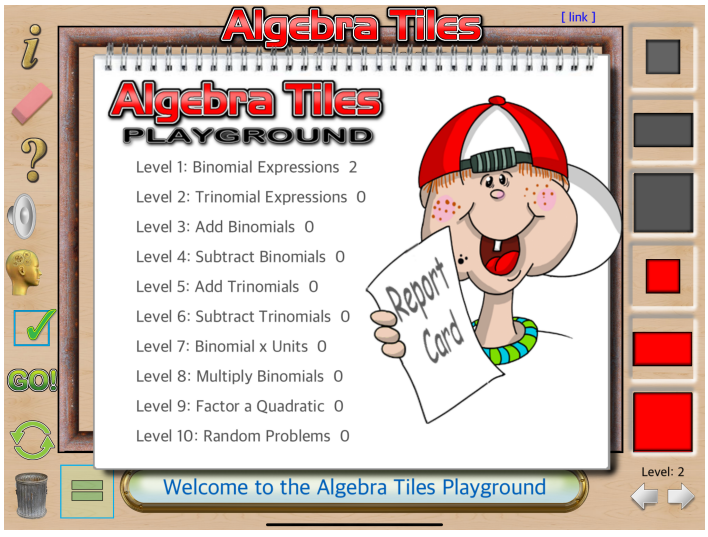
Special features make using the Algebra Tiles Playground fun:

1. The app helps students visualize abstract algebra concepts.
2. Binomials and Trinomials can be represented using Algebra Tiles.
3. Operations such as addition, subtraction, multiplication and factoring can be demonstrated using Algebra Tiles.
4. Speech and sound effects can be turned on or off.
5. The Algebra Tiles playground is available exclusively for Apple iPad.

The main instructional goals of the Algebra Tiles Playground app are given in these educational objectives:

1. To provide practice identifying and matching terminology related to introductory algebra. This app graphically represents terms and expressions, presenting key concepts of algebra in a way that motivates students to learn the associated algebraic ideas as well as processes used to simplify expressions.
2. To incrementally build an understanding of fundamental concepts by providing an easy-to-use format for exploring algebra.
3. To support the development of a student's sense of confidence in his or her mathematical ability by measuring progress toward achieving 10 specific goals.

Level 1: Binomial Expressions
Level 2: Trinomial Expressions
Level 3: Add Binomials
Level 4: Subtract Binomials
Level 5: Add Trinomials
Level 6: Subtract Trinomials
Level 7: Binomial x Units
Level 8: Multiply Binomials
Level 9: Factor a Quadratic
Level 10: Random Problems



Algebra Concepts covers the content recommended by national and state instruction resources. It supports the curriculum frameworks for California and other states. The content specifically follows the recommended guidelines provide by most of the state departments of education. Because an active learning approach is used, Algebra Tiles Playground also supports performance standards specific to the secondary math curriculum. The main topics covered are:

Variables and Expressions
Real Numbers and Their Properties
Solving Equations
Binomials and Polynomials
Factoring Polynomials

Constructivist Experiences

Algebra Tiles Playground is designed to provide a variety of learning experiences using different approaches and creative teaching methods. One key purpose for using the app is to provide students with constructivist experiences related to fundamental algebra concepts.

Cooperative Learning

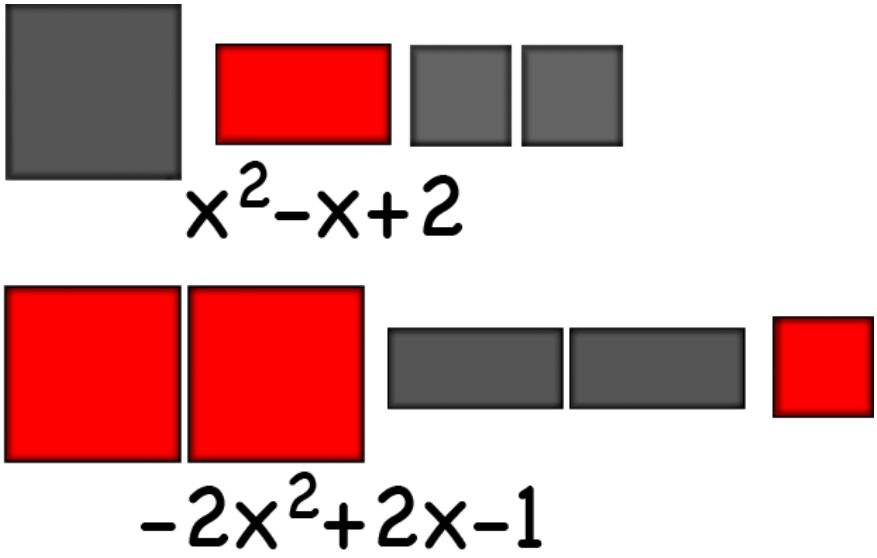
Algebra Tiles Playground provides an experience that is appropriate for individuals or small cooperative learning groups. Students can work together to share and compare their approaches to finding the solution to a specific set of algebra problems. These problems can be provided by the teachers or generated randomly by the app.

Learning Styles

Research in education has found that students have a variety of learning styles and modalities which must be considered when designing instructional approaches. Algebra Tiles Playground uses many techniques which are well-suited to visual learners. The special auditory features help with students who favor that modality. Algebra Tiles Playground is an effective way to make abstract algebraic ideas much more understandable. In addition, because interactive sound and speech are used at various points in the app experience, auditory learners will be more engaged. Kinesthetic learners are engaged through the use of the interactive experience with the iPad. Also, several of the activities suggested in the teachers guid and on the website involve a confluence of visual, auditory and kinesthetic learning approaches.

Independent Investigations

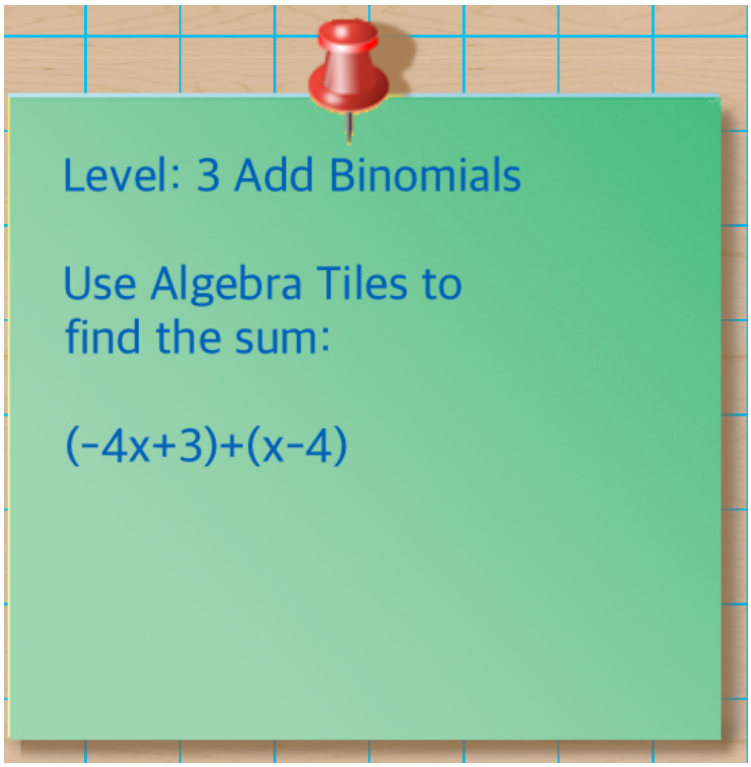
The lesson ideas and instruction activities provided in this teacher's guide as well as those in the app encourage students into independent investigations of topics related to the application of algebra concepts which are targeted by the scope of the program.



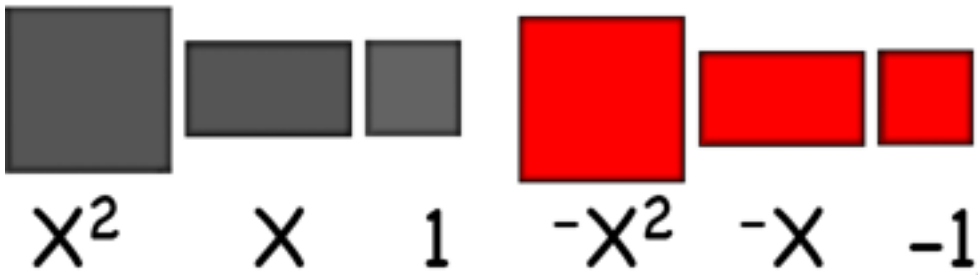
An Open-ended Approach

Providing for open-ended question in math may seem like an oxymoron when we consider that most problems have a single answer. However, giving students an opportunity to thing divergently in math class is important as a way to spark creativity and stimulate insight. Algebra Tiles Playground attempts to provide open-ended experience whenever possible. One example is from Level 3: Add Binomials. This problem is randomly generated by an algorithm in the app.

The challenge is to use the Algebra Tiles to find the sum. Begin by first building the expression (-4x+3) and then add (x-4)



This chart shows the meaning of each tile. Black tiles are positive and red tiles are negative. One way to remember this is to think of accounting where ‘in the black’ is positive and ‘in the red’ is negative.

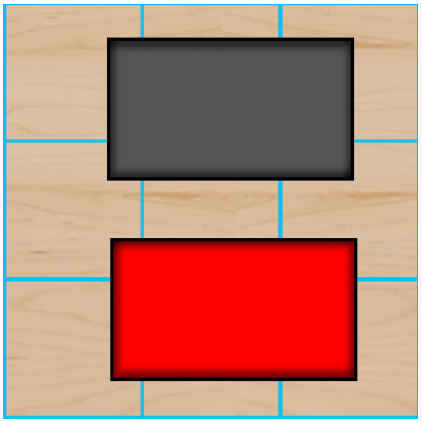


Critical Thinking

Algebra Tiles Playground engages the student in critical thinking and decision making through a series of related activities. A key feature in the design of Algebra Tiles Playground the grid area where tiles can be manipulated.

The grid area is smart in the sense that as tiles are placed on the grid, immediately that status of the tiles is given as an algebraic expression.

A critical part of learning algebra is to understand the identity property and how to use it. The identity for addition is 0. The identity for multiplication is 1. Show the identity for addition by dragging a +x and a -x onto the playground



Notice that the playground is evaluated as 0.

A Step-by-Step Example

Let's take a look at how to find the sum of (3x+2) and (2x-4).

1. Build the express 3x+2 on a blank playground.

2. Now add(2x-4) to the playground grid.

3. Use the green equal sign icon to create a special evaluation area on the playground grid.

4. The tiles in the green area are equal to zero (0) and therefore can be removed from the expression leaving (5x-4).

The Value of a Hands-On Approach

This type of instructional approach will help student who are first learning to work with operations involving polynomials internalize the underlying concepts and will better enable these students to generalize and make inferences about algebra problems that them encounter later in their high school and college academic careers. Internalizing fundamental concepts will facilitate a students ability to apply principles of algebra to new learning experiences.

Educators will find that Algebra Tiles Playground can be used in combination with other instructional programs and closely parallels the algebra curriculum taught in most secondary schools. In recent years the introduction of algebraic ideas has moved to younger grades and it is in these early years where a strong foundation can be built using virtual manipulative such as those found in the Algebra Tiles Playground.

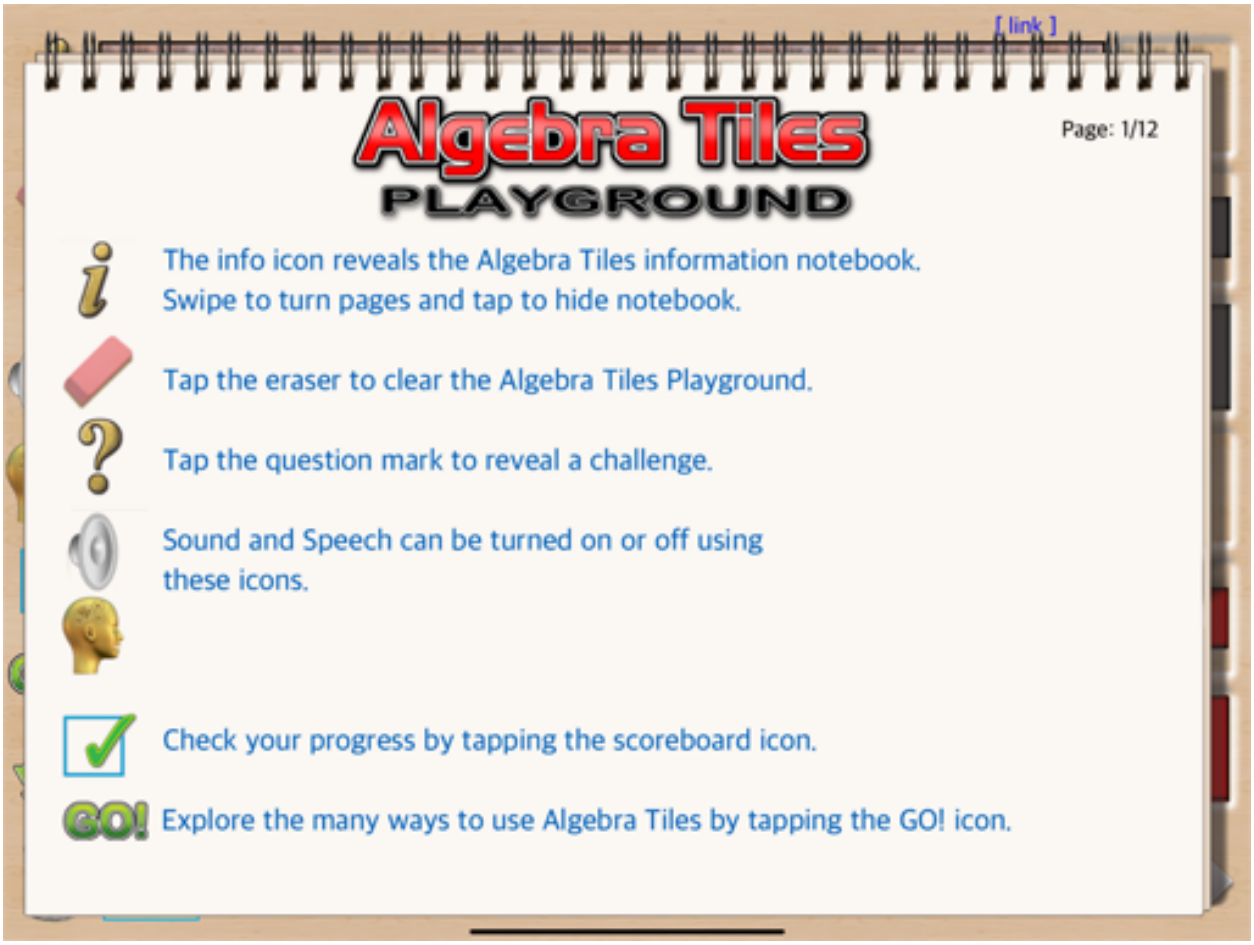
Algebra Tiles Playground provides students with the opportunity to make learning fun. With 10 levels of skill, the app should be useful for a wide range of students. Here, for example, a challenge is automatically generated and when successfully solved, the app responds tell the student the the solution has been found.

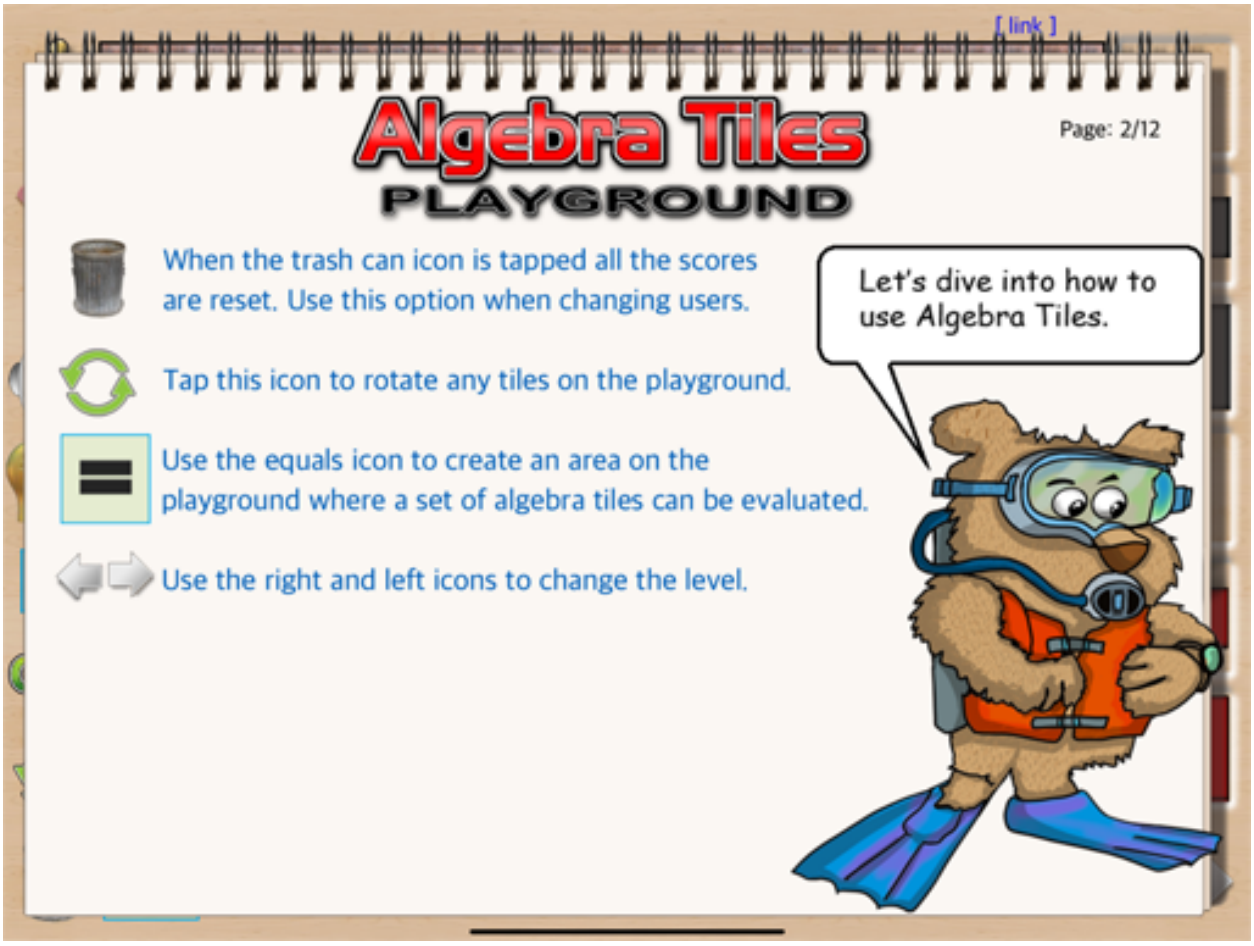
Algebra Tiles Playground is useful as a supplement for most introductory level algebra students. Much of the confusion that students feel is due to an inability to understand the terminology used to describe the fundamental rules of algebra. Algebra Tiles Playground is an effective tool for mastering the vocabulary of algebra. In a similar way, the program is effective for students who are also learning English because the synthetic speech feature can be used to narrate each action that the student does while using the app. Key words are pronounced by the iPad. Colorful images are used throughout the app and several other instruction strategies are employed to help all students succeed.

With Algebra Tiles Playground students enjoy learning new ideas that are essential for success in high level math classes. The app is an effective way to introduce students to algebraic terms and fundamental concepts. It is also a good way to provide reenforcement and to strengthen skills.

The supplementary materials that provided in this manual are designed to be used in conjunction with the app. The supplementary worksheets can be duplicated or teacher may find it useful to project the worksheet with a presentation system in the classroom.

Here is a list of the icons used in the program and their functions. Swipe from the right to the left to get to the next notebook page and swipe from the left to the right to go to the previous page.





Terms Algebra Students Should Know:

Absolute value
Adding to both sides
Addition identity property
Addition of opposites
Associative property
Base
Binomial
Brackets
Closure property
Coefficient
Commutative property
Distributive property
Equal
Exponent
Exponent form
Exponents in multiplication
Factored form
Fraction bar
Graph of a number
Greater than
Greater than equal to
Less than
Less than equal to
Monomial
Multiplication identity
Multiplication of equals
Multiplying both sides
Multiplying by a reciprocal
Negative integers
Negative integers
Not equal
Opposite of a sum
Opposites
Opposites in products
Order of operations
Origin
Parentheses
Positive integers
Power of a power rule
Power of a product rule
Property of equality
Property of negative one
Property of zero
Replacement set
Rule for division
Rule for subtraction
Simplification
Simplifying an expression
Trinomial
Value of a variable
Variable

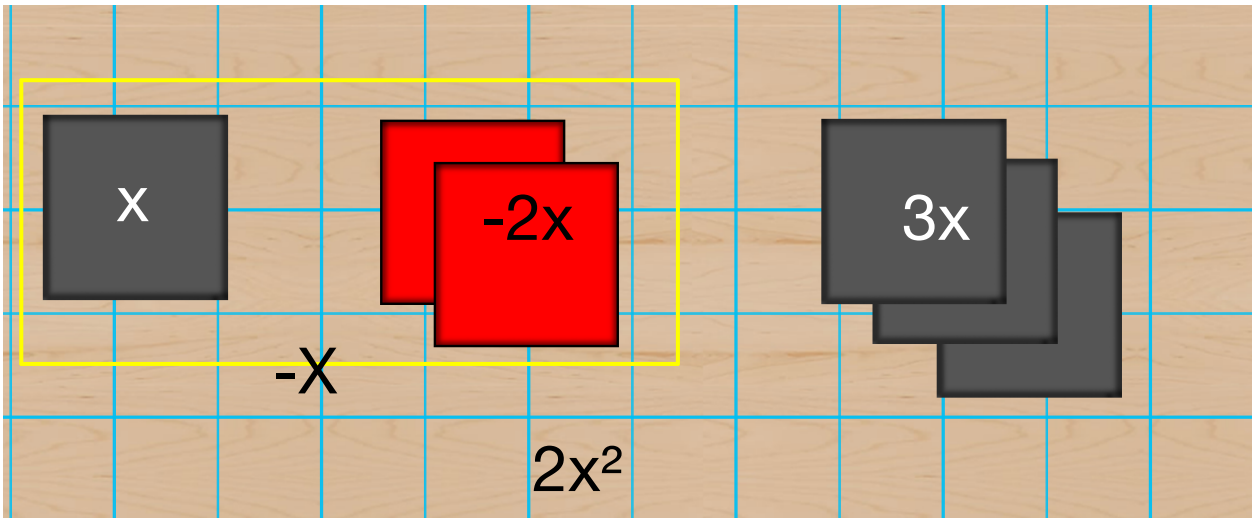
The Associative Property

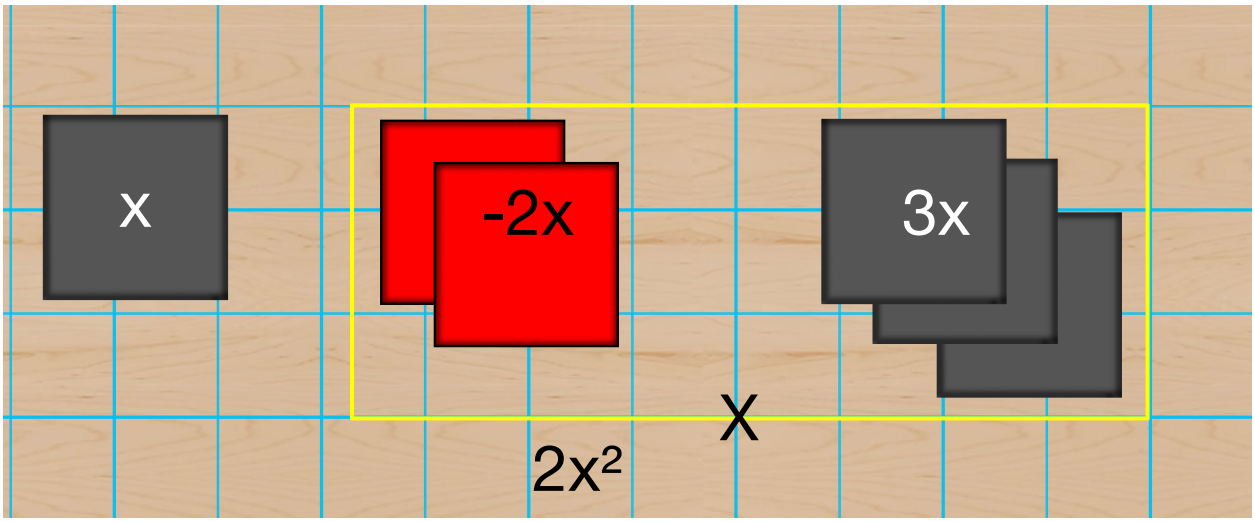
In Algebra, the associative property is a rule that states that when you are adding or multiplying numbers, it does not matter how the numbers are grouped, meaning it doesn't matter where you put the parentheses.

Here is an example:

$$(a+b)+c=a+(b+c)$$

Visualize the Associative Property with Algebra Tiles:





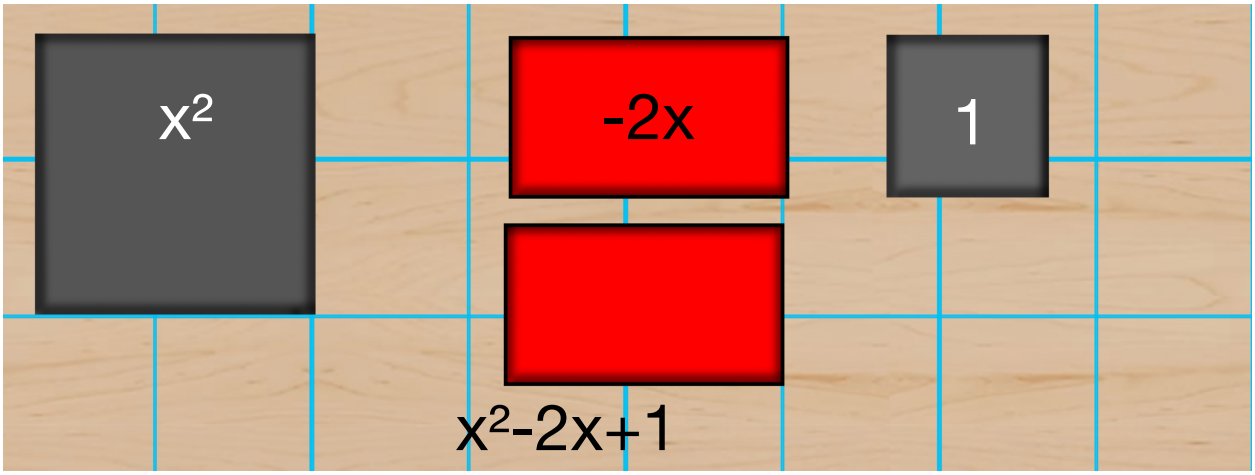
Commutative Property

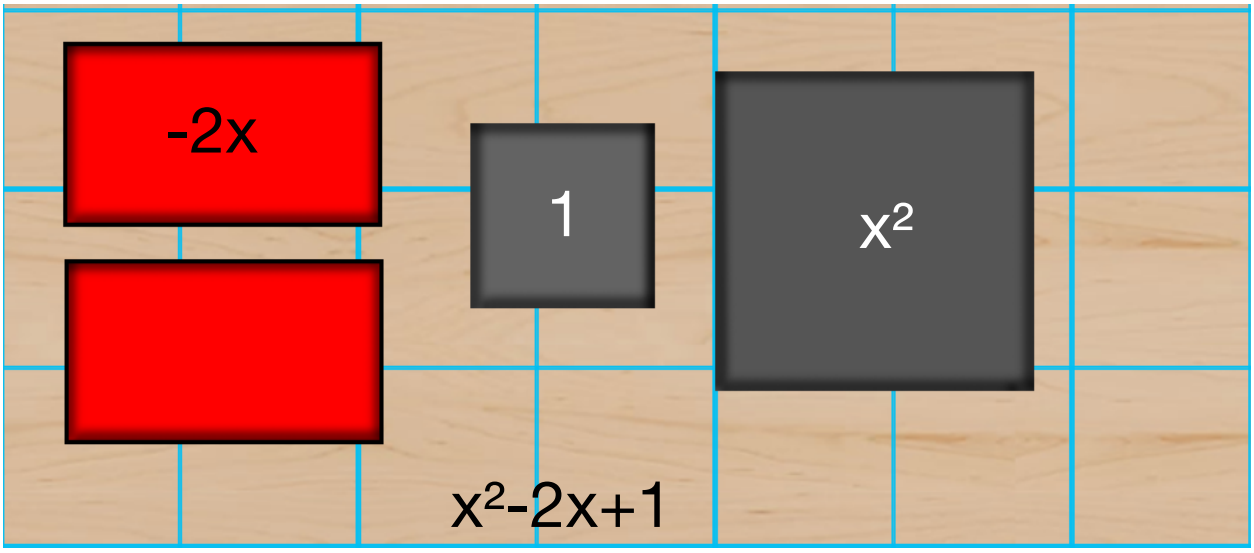
In Algebra, the commutative property is a rule that states that when you are adding or multiplying numbers, it does not matter which order the numbers are multiplied. In other words, changing the order in which numbers are added or multiplied does not change the result.

Here is an example:

$$a+b+c=c+b+a$$

Visualize the Commutative Property with Algebra Tiles:



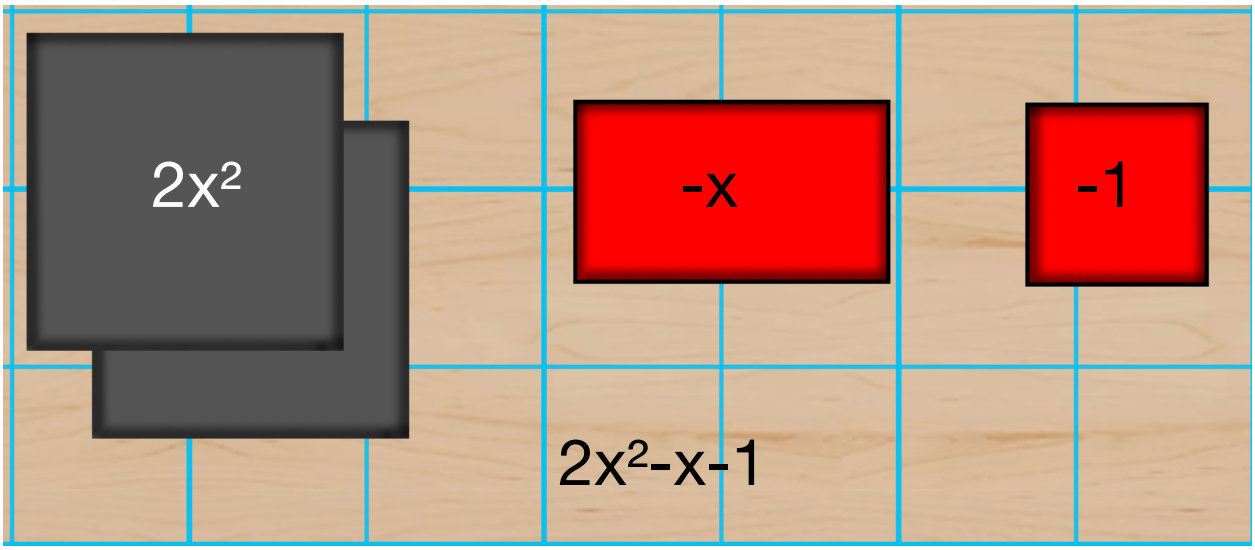


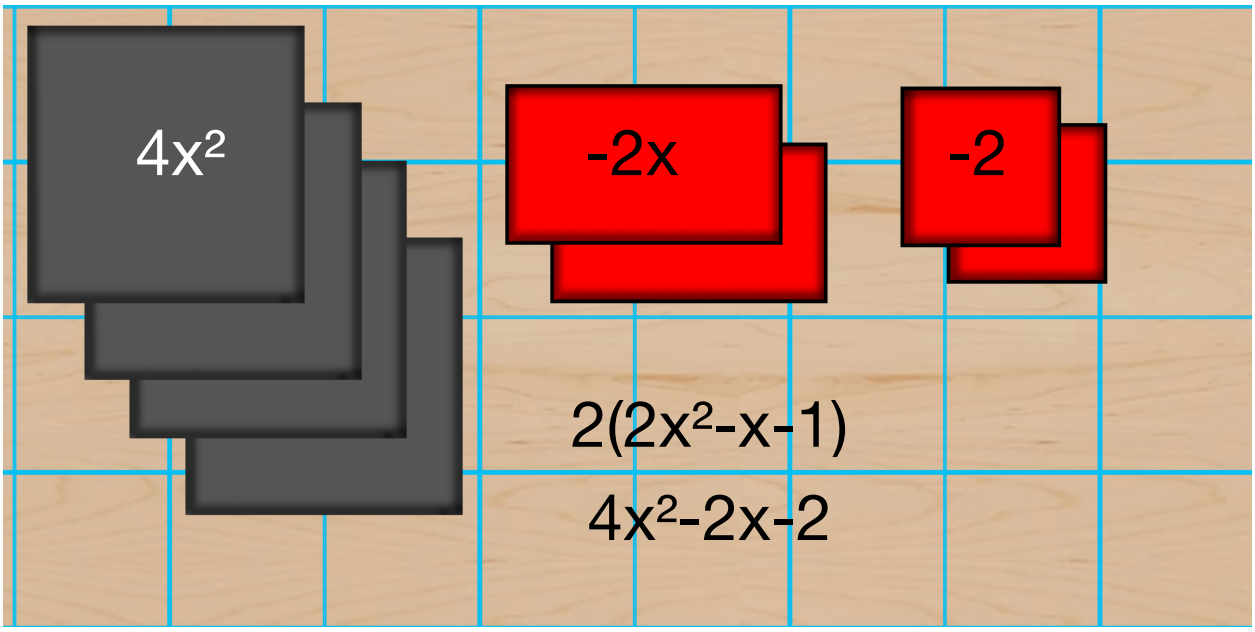
Distributive Property

In Algebra, the distributive property is a rule that explains how to simplify an expression written in the form a(b+c). In other words, all the terms within the parentheses are multiplied by the a term.

Here is an example:

$$a(b+c)=ab+ac$$





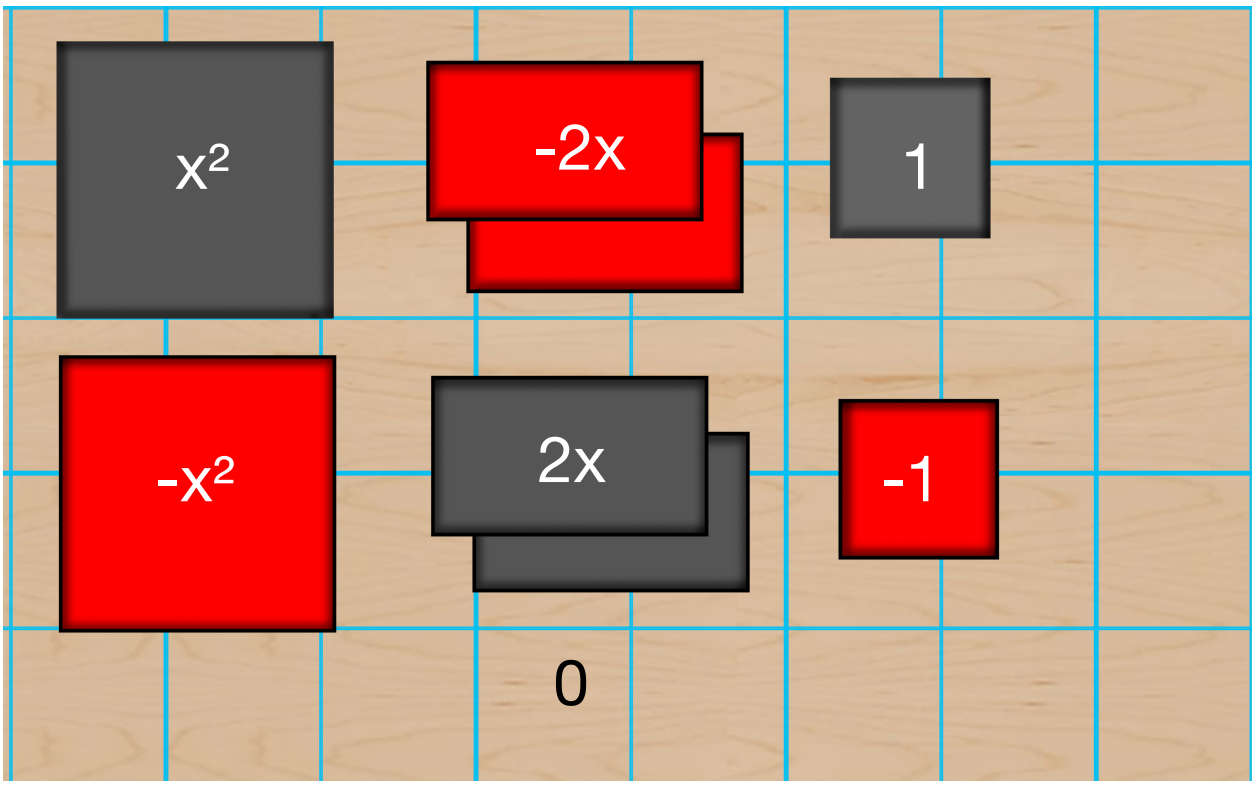
Addition of Opposites

In Algebra, adding two opposites results in zero (0).

Here is an example:

$$a+(-a)=0$$

Using Algebra Tiles we can show that $(x^2-2x+1)+(-x^2+2x-1)=0$.

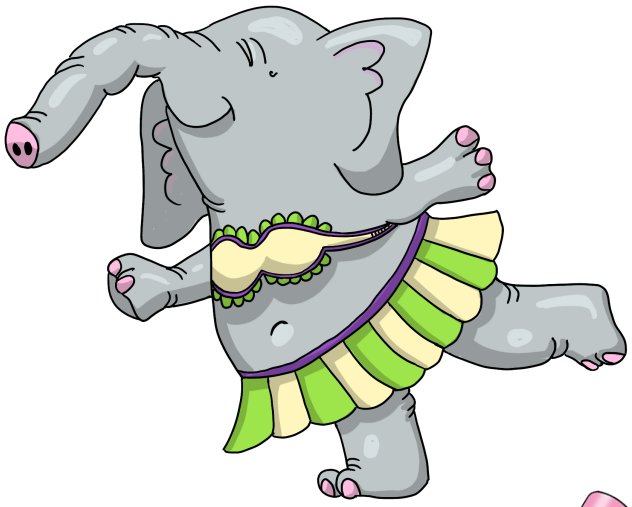


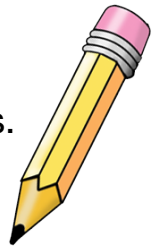
Evaluating Expressions

This table shows the values of six variables. Substitute the value of the variable given to

Values of the Variables

a=3	x=10
b=5	y=2
c=8	z=1





Find the value of each expression.

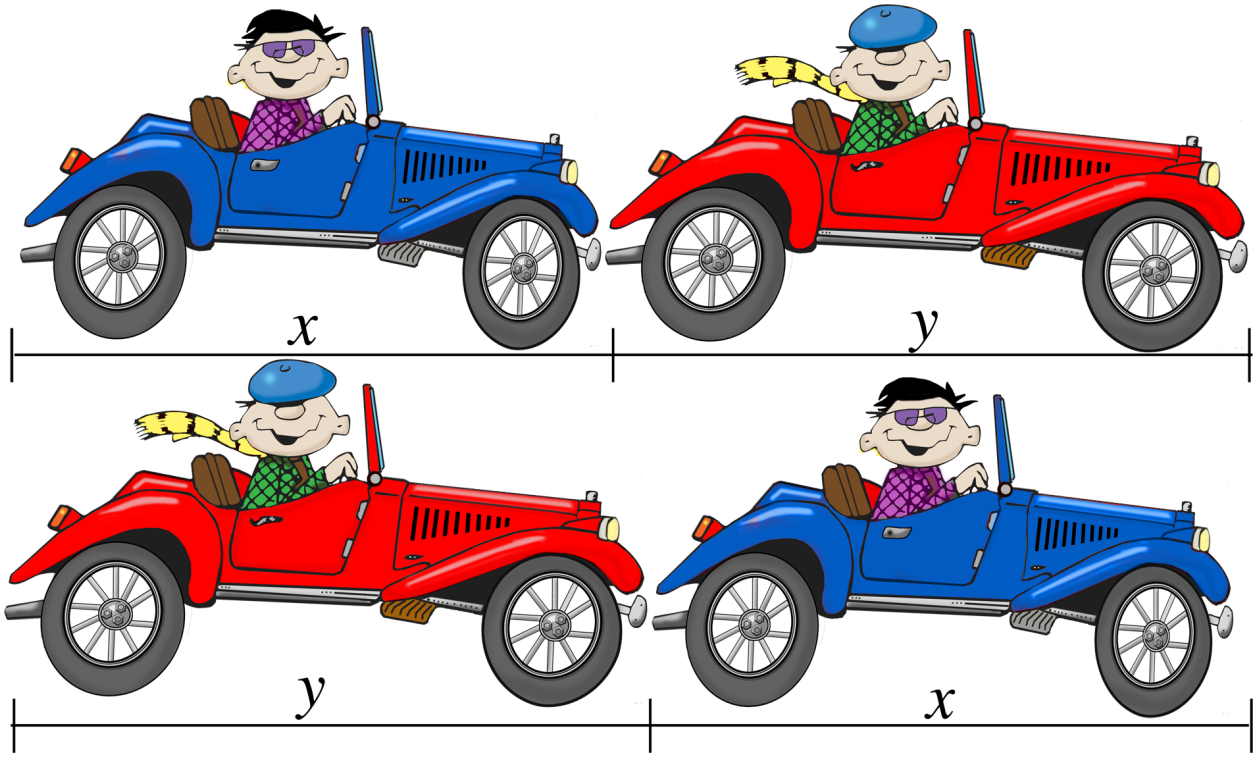
1. 3y+4
2. 4z+3
3. Xyz
4. x+y+z
5. 4a-y
6. 3b+z
7. (x+y)
8. 2x+3y+4z
9. 3y+4c+1
10. 4a+xy
11. 2y+4abc
12. 8ab-c

Simplify these expressions.

1. (7-3)+3+2
2. 5+(12-3)
3. 8+(3-1)+5
4. 1+2(4+6)
5. 3(5-2)
6. 3(2_6)-3
7. 3(5+2)
8. 6+(2+8)
9. 10-3(4-2)
10. 4+3(4-6)
11. 9-2(2+4)
12. (10-2)+3

Using the Commutative Property

Use the commutative property to write an equivalent expression.



Write an equivalent expression.

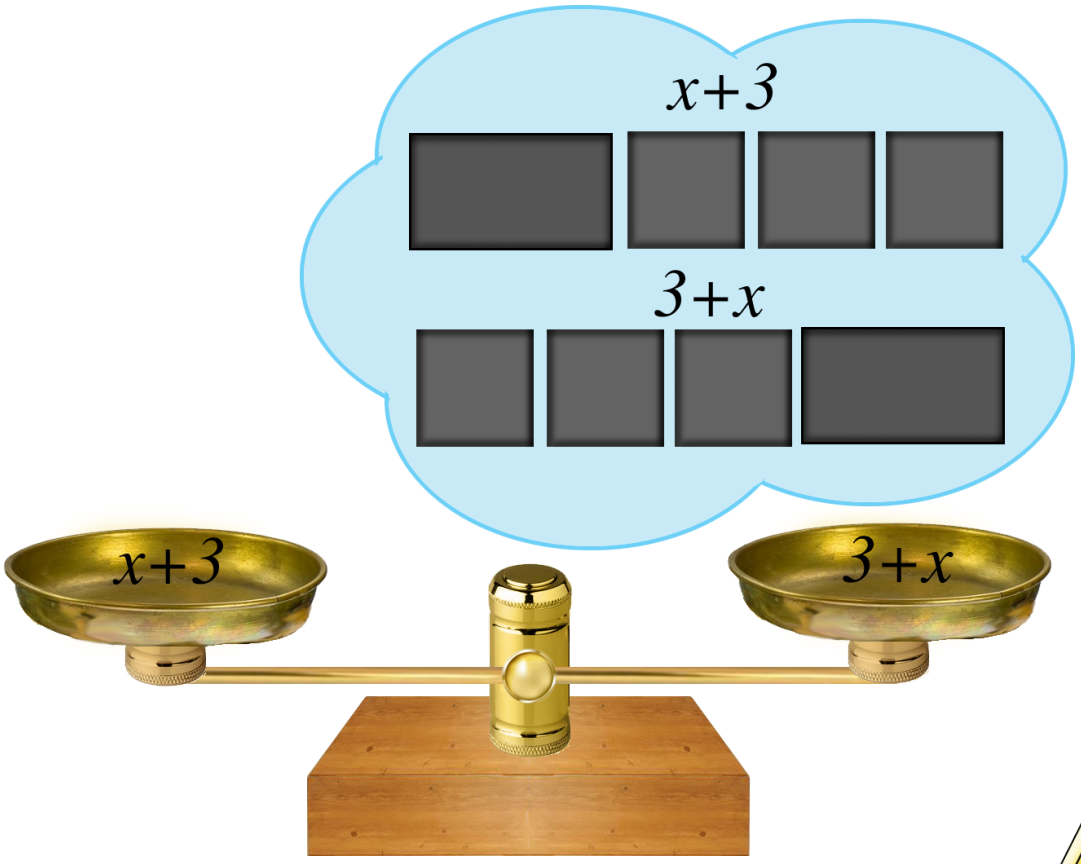
1. 3+x
2. 4y+9
3. 10x+3y
4. 2mn
5. 3mx+mn
6. 5a+2b
7. 8bc
8. abc
9. 9m+6c
10. 2x+y
11. xy+z
12. (3)(6)

Which of these are not equivalent?

1.	3y+2 and 2+3y	(yes)	(no)
2.	2x+3y and 3x+2y	(yes)	(no)
3.	2ab and ba2	(yes)	(no)
4.	2a+3b+4c and 9abc	(yes)	(no)
5.	ab(x+2) and (x+2)ab	(yes)	(no)
6.	x+y(a+b) and y+x(a+b)	(yes)	(no)
7.	3a+b and 3b+a	(yes)	(no)
8.	a+b+c and c+a+b	(yes)	(no)
9.	ab+(c-3) and (c-3)-ab	(yes)	(no)
10.	abc+d and d+abc	(yes)	(no)
11.	a(x+1) and ax+a	(yes)	(no)
12.	b(c-1) and bc+b	(yes)	(no)

Evaluating Expressions For Equality

Study each example to determine if the expressions are equivalent. Mark 'YES' if the statements are equivalent and 'NO' if they are not.



1.	3x+y and 3y+x	(yes)	(no)
2.	mn+1 and 1+mn	(yes)	(no)
3.	a+3b an 3b+3a	(yes)	(no)
4.	8+y and 8y+1	(yes)	(no)
5.	5m+y and my+5	(yes)	(no)
6.	3ab(2c) and 6abc	(yes)	(no)
7.	8a+2a+b and 10a+b	(yes)	(no)
8.	abc-c and ab	(yes)	(no)
9.	3(2a+b) and 6ab	(yes)	(no)
10.	5(ax)+ax and 6ax	(yes)	(no)

Problem Solving Using Algebraic Expressions

Solve each problem by substituting the values and evaluating each expression.

The area of a rectangle is the product of the length and the width.

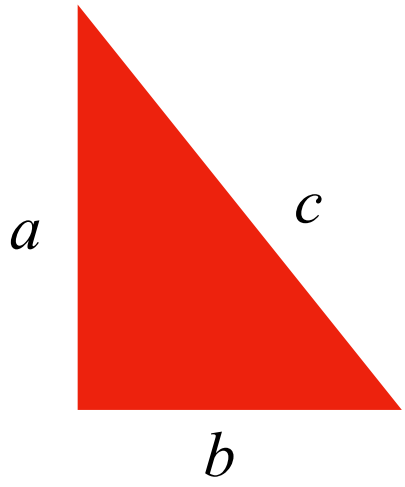
$$A=lw$$

1. If l=20 and w=15, find A.
2. If l=4 and w=8, find A.
3. If l=5.6 and w=3.5, find A.
4. If l=8.5 and w=9.0, find A.
5. If l=5 and w=3.5, find A.
6. If l=8.2 and w=9.3, find A.

The area of a triangle is by dividing the product of the length and width by 2.

$a=w;\ b=l \qquad A=\frac{ab}{2}$

1. If a=20 and b=15, find A.
2. If a=4 and b=8, find A.
3. If a=5 and b=8, find A.
4. If a=8 and b=2.5, find A.
5. If a=5 and b=3.2, find A.
6. If a=8.5 and b=4.7, find A.



The perimeter of a rectangle is found by multiplying the sum of the length and width by 2.

$$P=2(l+w)$$

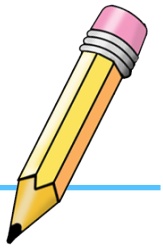
1. If l=25 and w=19, find P.
2. If l=4 and w=8, find P.
3. If l=4 and w=9, find P.
4. If l=8 and w=2.5, find P.
5. If l=6 and w=3.5, find P.
6. If l=8.5 and w=4.7, find P.

Finding the Perimeter of a Triangle

The perimeter of a triangle is the sum of the three sides.

$$P=a+b+c$$

1. If a=5, b=10 and c=3, find P.
2. If a=4, b=0.8 and c=1.5, find P.
3. If a=5.2, b=1 and c=3, find P.
4. If a=8, b=3.4 and c=2.5, find P.
5. If a=1.5, b=3 and c=1, find P.
6. If a=6.5, b=8.4 and c=4.3, find P.



Evaluate each expression. Use these values for the variables.

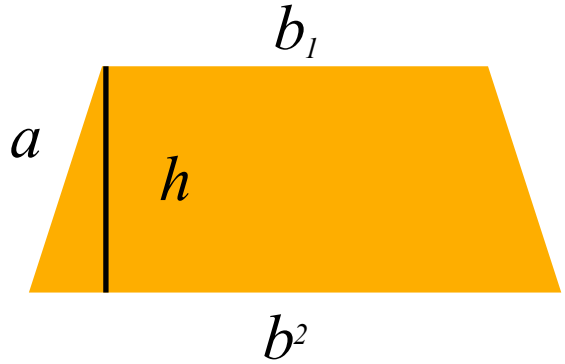
d=4 e=4 f=4

u=4 v=4 w=4

1. def
2. 3d+4u
3. w-(e+v)
4. 3e+4v
5. fw-3
6. 4d-(e+f)
7. u+e
8. w(d+2e)
9. 3w-(u+e)
10. 3e+4w

Finding the Area and Perimeter of an Isosceles Trapezoid.

Use these values to find the area and perimeter of an isosceles trapezoid.



$$P=2a+b_1+b_2$$

$$A=\frac{(b_1+b_2)}{2}h$$

1. a=4, b_1=8, b_2=5
2. a=6.5, b_1=10, b_2=8
3. a=10, b_1=9, b_2=9
4. a=3.5, b_1=8, b_2=4
5. a=3, b_1=9, b_2=8
6. a=3.5, b_1=12, b_2=10
7. a=10, b_1=9, b_2=6
8. a=4.5, b_1=5, b_2=5

Finding the Perimeter and Area of a Parallelogram

Use these values to find the perimeter of a parallelogram.

$$P=2(a+b)$$

$$A=bh$$

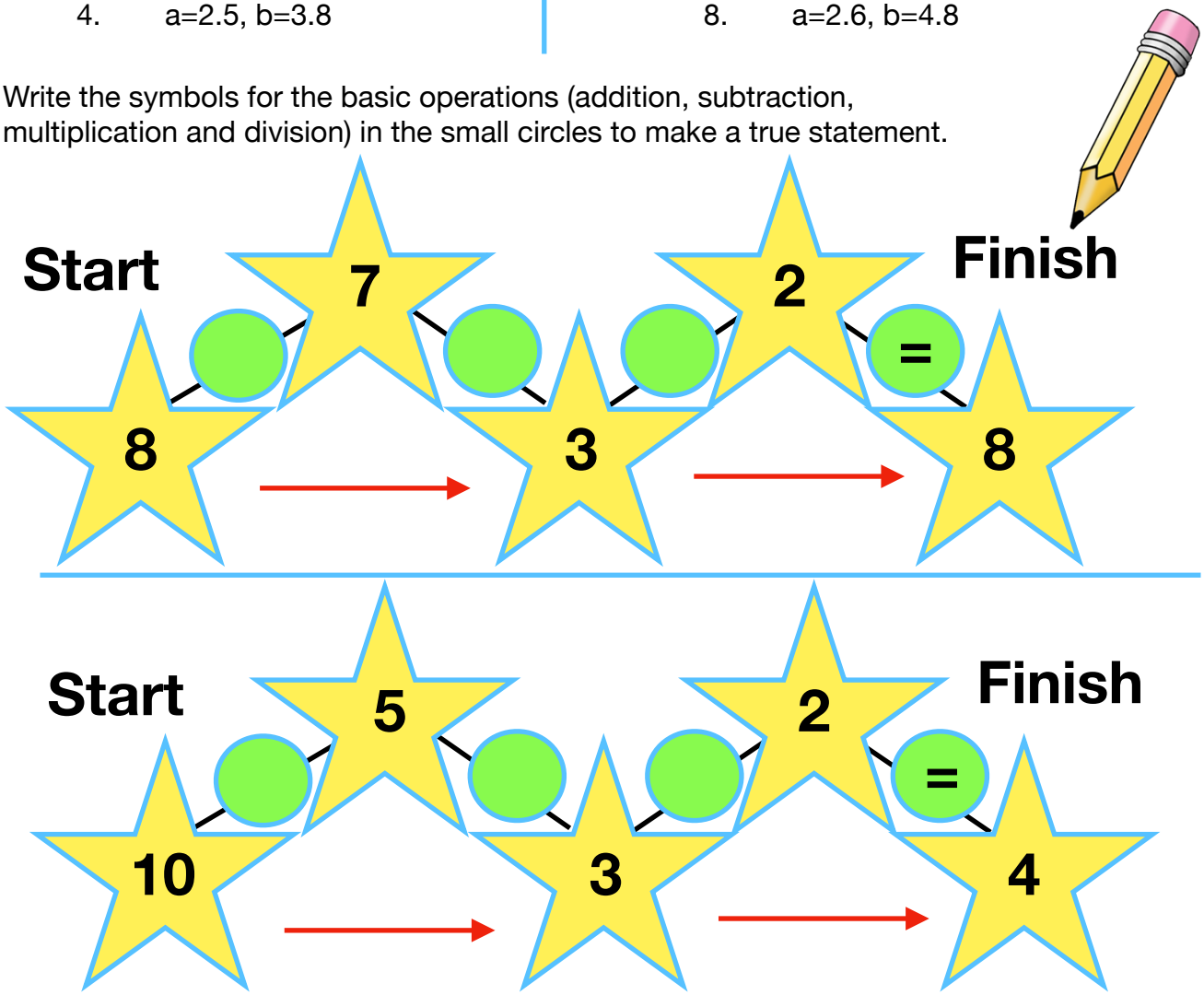
Find the perimeter of a parallelogram with these dimensions.

1. a=3, b=5
2. a=3.5, b=4.5
3. a=10, b=6
4. a=2.5, b=3.8

Find the area of a parallelogram with these dimensions.

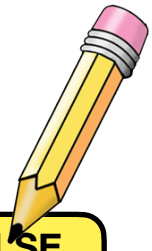
5. a=7, b=5
6. a=5.5, b=4.5
7. a=10, b=5
8. a=2.6, b=4.8

Write the symbols for the basic operations (addition, subtraction, multiplication and division) in the small circles to make a true statement.



Comparing Expressions

Use these values to find the perimeter of a parallelogram.



1. $3 < 1 / \left(\frac{3}{5}\right)$ TRUE FALSE

2. $\frac{3}{4} \times \frac{2}{3} < 1$ TRUE FALSE

3. $\frac{(10+(4-2))}{9-3} > 1.4+3.8$ TRUE FALSE

4. $\frac{3.6+(3-1)}{3.5+.5} > 3.9$ TRUE FALSE

5. $\frac{3}{4} \times \frac{1}{3} > \frac{4-3}{2 \times 3}$ TRUE FALSE

6. $\frac{(10-3)}{4 \times 2} > 0.875$ TRUE FALSE

7. $3(4+2) > 7(2+1)$ TRUE FALSE

Using Number Properties to Simplify Expressions

The operations of addition and multiplication have several important properties. These properties are basic methods that are used to simplify expressions. The properties are assumed to be true and are called axioms or postulates.

The Axiom of Closure: If ***a*** and ***b*** are real numbers, that ***a+b*** is a unique real number and ***ab*** is a unique real number.

Simplify each expression to find a unique real number. Show your work.

1. $(34+23)(12+10)$

2. $2\frac{3}{8}\times 1\frac{3}{4}$

3. $3.45+12.8$

4. $9\times 8+17$

5. $\frac{3}{8}+\frac{1}{5}$

6. $3\frac{1}{4}\times 9\frac{1}{2}\times 2.5$

7. $1700+2349$

8. 1934×27

9. $1\frac{1}{4}+(2\frac{1}{3}\times 3\frac{1}{4})$

The Commutative Property: if ***a*** and ***b*** are real numbers, then ***a+b*** is equal to ***b+a*** and ***ab*** is equal to ***ba***.

Usually addition and multiplication are performed in a left to right order.

33+20+14+78+21+98+78

The Associative Property: if ***a, b*** and ***c*** are real numbers, then ***(a+b)+c*** is equal to ***a+(b+c)*** and ***(ab)c*** is equal to ***a(bc)***.

$$(3s+2t)+5t=3s+(2t+5t)=3s+7t$$

$$(4x)(5a)(6z)=4\times 5\times 6axz=12-axz$$

Some problems are easier to compute if the terms are in a different order or a grouped in a different way. For example the expression on the left is easier to compute if it is changed to the expression on the right.

$$25 + 47 + 25 \qquad 25 + 25 + 47$$

And in this multiplication, the expression on the right is easier to calculate than the one on the left.

$$(38 \times 50) \times 3 \qquad 38 \times (50 \times 2)$$

Use the commutative and associative properties to write each of these expression in way that is easier to compute. Simplify the expression.

1. $50 + 98 + 25 + 34 + 25$

2. $(23 \times 25) \times 4$

3. $\frac{1}{3} + \frac{5}{8} + \frac{2}{3}$

4. $3\frac{1}{4} + 2\frac{3}{5} + 2\frac{3}{4}$

5. $2.5x(6.7 \times 4)$

6. $60 + 45 + 35 + 40$

Use the commutative and associative properties to simplify these expressions. For example:

$$(3a)(4b) = (3 \times 4)(ab) = 12ab$$

1. $(4a)(9b)(5c)$

2. $3x + 6y(9z) + 2x$

3. $(3b)(9a)(2b)$

4. $\frac{((34 + 2s)8r)}{(5t)a(3m)}$

5. $6b(4a(3b))$

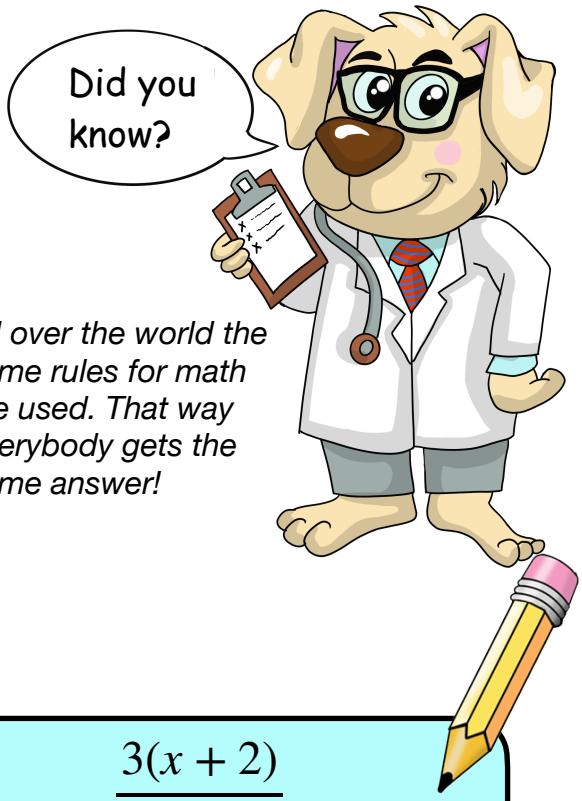
6. $(3x4y)(y^2))2x$

Working with Parentheses, Brackets and Fraction Bars

Parentheses, brackets and fraction bars are used to tell the order in which calculations should be performed. Study these examples:

Simplification Process

1. $(-3+2)8a$
2. $-1(8a)$
3. $-8a$



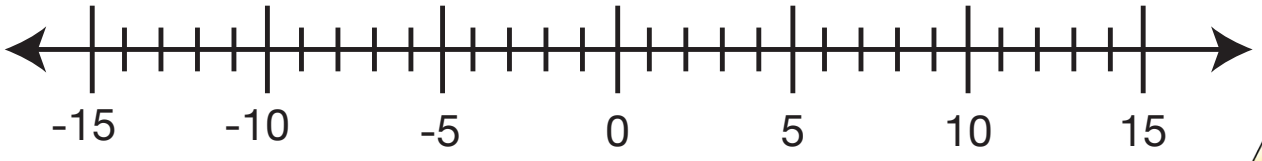
All over the world the same rules for math are used. That way everybody gets the same answer!

Simplify each expression. Remember to do the operations within the parentheses and brackets first. Multiply or divide in a left-to-right order second and do addition and subtraction last.

1. $[(3 \cdot 4)+5]+6$
2. $2(3x+4x)$
3. $(2\frac{1}{2}+3\frac{1}{3})7$
4. $(-5(-4)+3)+10$
5. $5x[3x+(2x+4x)]$
6. $\dfrac{3(x+2)}{x+2x}$
7. $\dfrac{1-(x+2)}{4(x-2)}$
8. $\dfrac{15(12-9)}{(4.55+4.45)}$
9. $\dfrac{((2+9)-5)12}{6(18-15)}$
10. $[(3\frac{1}{5}+4\frac{1}{10})5] \div (1\frac{3}{4}+3\frac{1}{4})$

Locating Points on a Number Line

Every real number corresponds to a point on a number line. Simplify each expression and locate the result on this number line.



A. $\dfrac{(-3-(-3))}{(4-2)}$

B. $\dfrac{((9+3)+(7-(9-4)))}{(3^2-2)}$

C. $(-1(-5)+2)2$

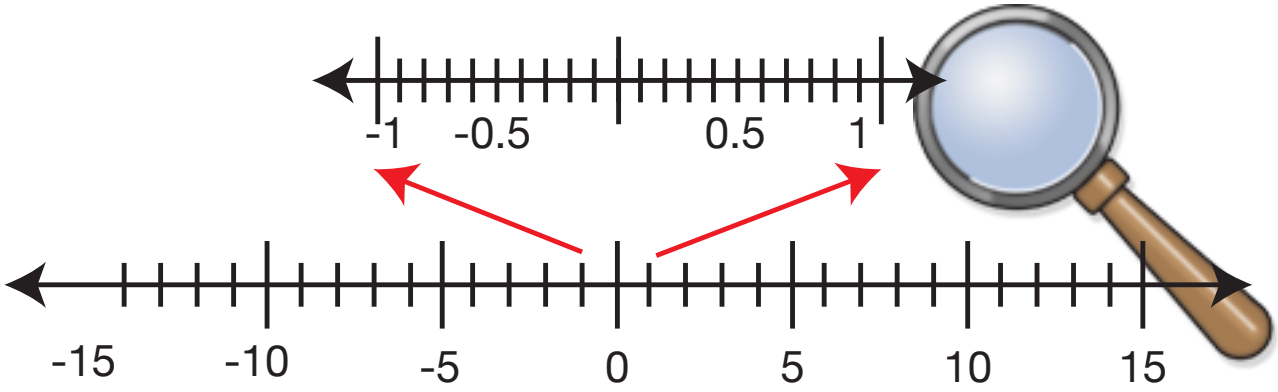
D. $(-|5|-3)$

E. $(-5+3)9)+69$

F. $1-(-1(1+2))$

G. $(5+(7-(9-4)))$

H. $-(-8)+2$



1. $|3-2.5|$

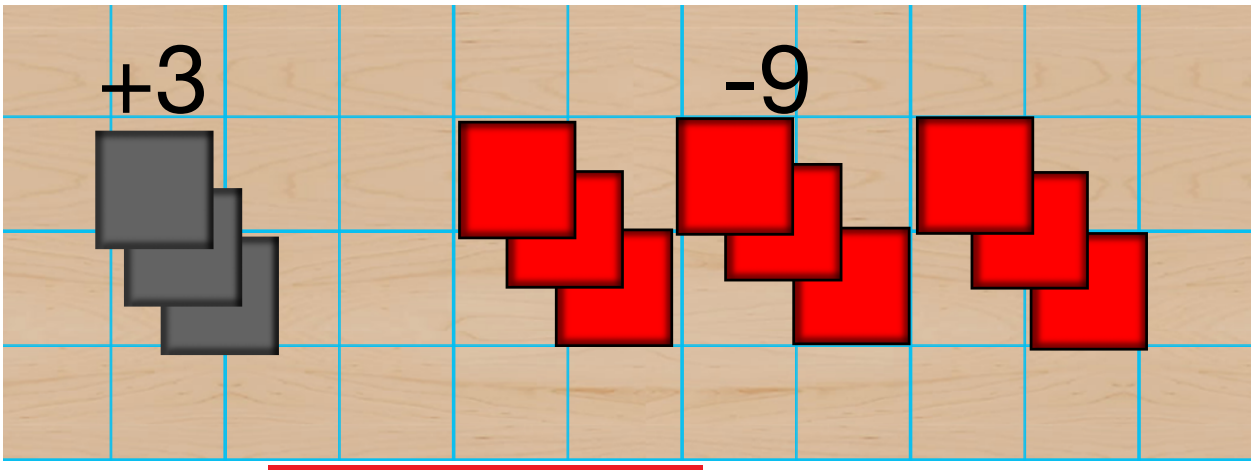
2. $\dfrac{|8-(3+4)|}{|5-(2+1)|}$

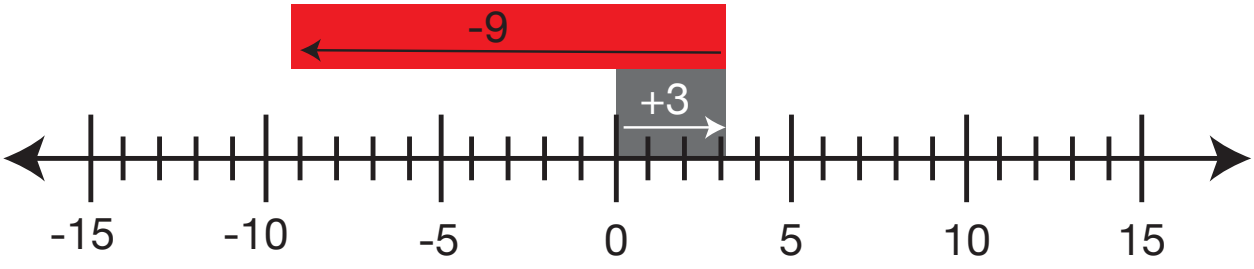
3. $-|0.1-0.3|$

4. $2^2-3.5$

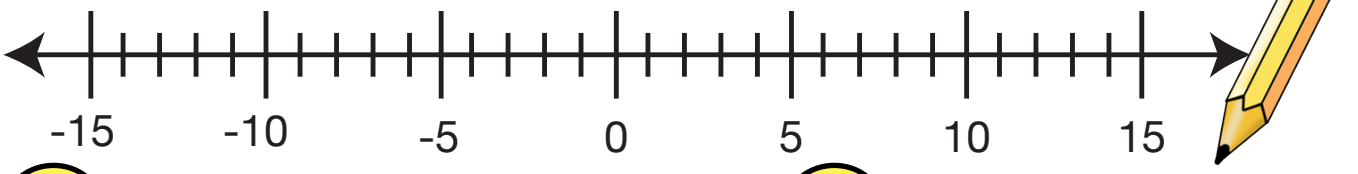
Adding Positive and Negative Numbers

Adding a positive can be thought of as moving to the right on a number line. Moving to the left on a number lines the same as adding a negative number. To find the some of (+3) plus (-9) start at +3 on the number and move to the left 9 units. The result is (-6). To find the absolute value of a negative number using the number line local the positive number that is exactly the same distance from zero (0).





Use this number line to simplify each expression.

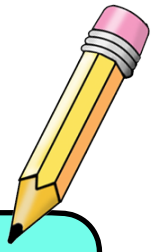


1. $(-2+8)+(-4)$
2. $-6+(-3-2)$
3. $(-15+22)+(-3)$
4. $-3\frac{1}{2}+(-\frac{1}{4})$
5. $-4.5+(-3.2)$
6. $-1+(-3)-4+2$
7. $-6+|-7|$
8. $\frac{|-4|}{5+(-3)}+(-5)$
9. -3+|-7|
10. |-3+8|

Working with the Distributive Property

The distributive property is used in simplifying expressions. The rule states that for all real numbers a, b and c:

$a(b+c) = ab + ac$ and $(b+c)a = ba + ca$



Use the distributive property to simplify these expressions:

1. $30(\frac{1}{3} + \frac{2}{5})$

2. $3a(5b + 8c)$

3. $3(\frac{1}{3}a - b)$

4. $-18t + 12t$

5. $15s + (-3)s$

6. $6a + 7a$

7. $3(b + 7) + 8$

8. $8(x + 3y) - 14$

9. $10z + 5(b + z)$

10. $a(b + c)$

11. $b(3bc - c)$

12. $3a(b + 2c)$

13. $4(2ab - 3c)$

14. $2(a - 3) + 5b$

Reflecting on Reciprocals

Each number in the set of real numbers has a reciprocal except for zero (0).

$$a(\frac{1}{a}) = 1 \qquad \frac{1}{a}(a) = 1$$

Study this examples of reciprocals.

$3(\frac{1}{3}) = 1$ $\quad \frac{5}{8}(1\frac{3}{5}) = 1$ $\quad -0.5(-2) = 1$

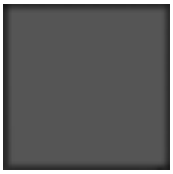
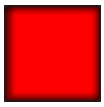
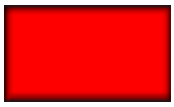
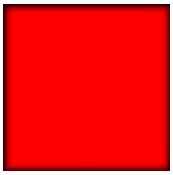
$0.4(2.5) = 1$ $\quad xyz(\frac{1}{xyz}) = 1$ $\quad \frac{1}{ab^2}(ab^2) = 1$

Draw a line to match each expression on the left with its reciprocal on the right.

$\frac{a}{bc^2}$	⬤	⬤	$\frac{ab}{c^2}$
$(ab + 3c)$	⬤	⬤	$\frac{1}{a + bc}$
$a(\frac{b}{c})$	⬤	⬤	$\frac{b}{2a}$
$a + bc$	⬤	⬤	$\frac{bc^2}{a}$
$\frac{2a}{b}$	⬤	⬤	$\frac{c}{ab}$
$\frac{c^2}{ab}$	⬤	⬤	$\frac{1}{ab + 3c}$

Algebra Tiles Playground: An Introduction

Algebra Tiles are used to represent positive and negative values.

	1	x	x^2
Positive			
Negative			

Algebra Tiles Playground is designed to be an interactive learning tool. Here is an explanation of the meaning of the icons and their related functions:

The info icon reveals the Algebra Tiles information notebook. Swipe to turn pages and tap to hide notebook.

Tap the eraser to clear the Algebra Tiles Playground.

Tap the question mark to reveal a challenge.

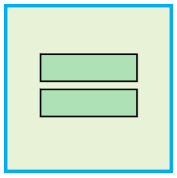
Sound and Speech can be turned on or off using these icons.

Check your progress by tapping the scoreboard icon.

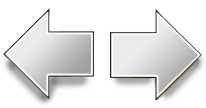
Explore the many ways to use Algebra Tiles by tapping the GO! icon

Tap this icon to rotate any tiles on the playground.

When the trash can icon is tapped all the scores are reset. Use this option when changing users.



Use the equals icon to create an area on the playground where a set of algebra tiles can be evaluated.



Use the right and left icons to change the level.

Algebra Tiles Playground: Instructional Levels and Features

Algebra Tiles Playground can be used to explore and learn about these important topics:

Level 1: Binomial Expressions

Level 2: Trinomial Expressions

Level 3: Add Binomials

Level 4: Subtract Binomials

Level 5: Add Trinomials

Level 6: Subtract Trinomials

Level 7: Binomial x Units

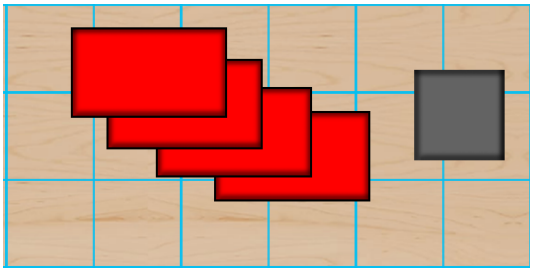
Level 8: Multiply Binomials

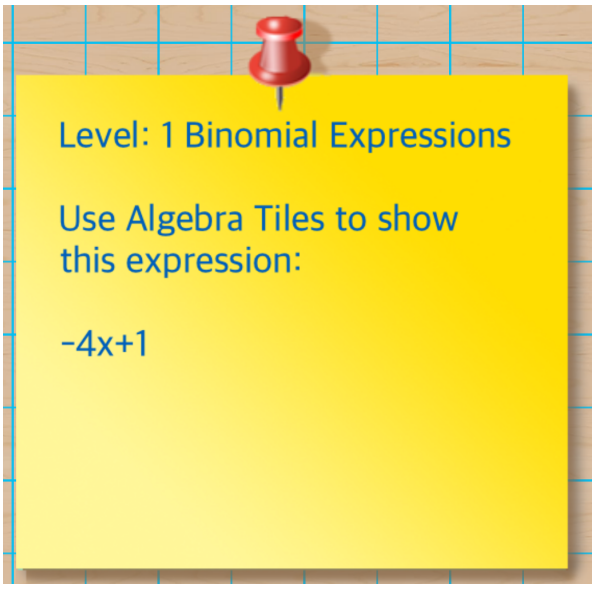
Level 9: Factor a Quadratic

Level 10: Random Problems

Use the right and left arrow icons to select the level.

Tap the question mark to reveal a challenge. The challenges appear on a note that can be moved around the screen. Answer the question by sliding algebra tiles into the playground area. For this question the answer would be:

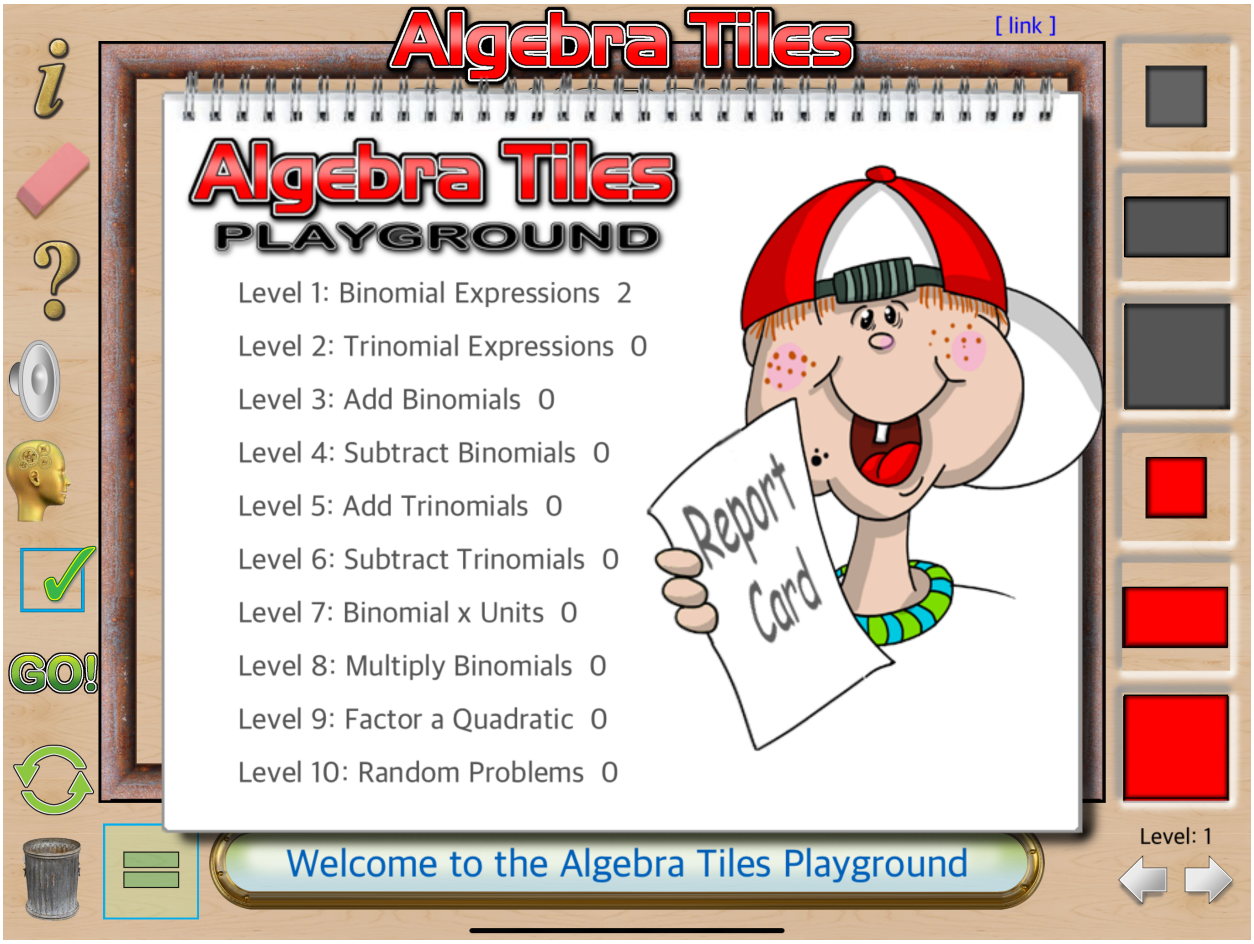




Remove the note from the screen by sliding it to the side.

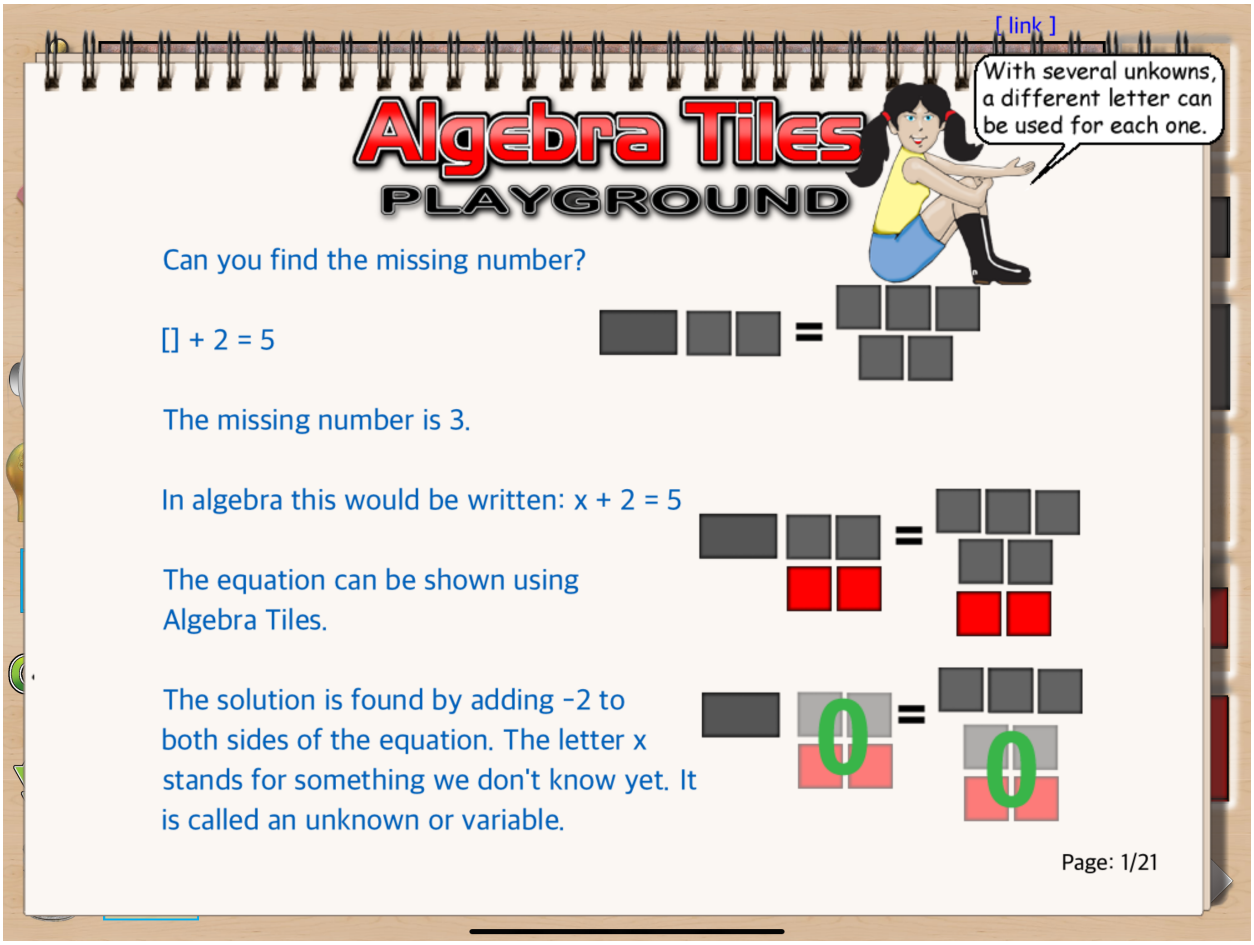
As the student works through the randomly generated problems at a particular level a record of their scores.

Check your progress by tapping the scoreboard icon.



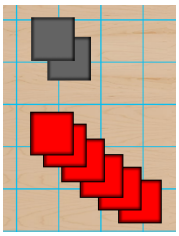
Tap the trash can icon to reset the scores by setting them all to zero.

A lesson is available in the app. It explains how the app is used as well as how to work with Algebra Tiles. Tap the GO! icon to start the lesson. It appear in a ring bound notebook. Tap anywhere outside the notebook to exit the lesson.



Use the Algebra Tiles Playground to solve these problems. Here's an example.

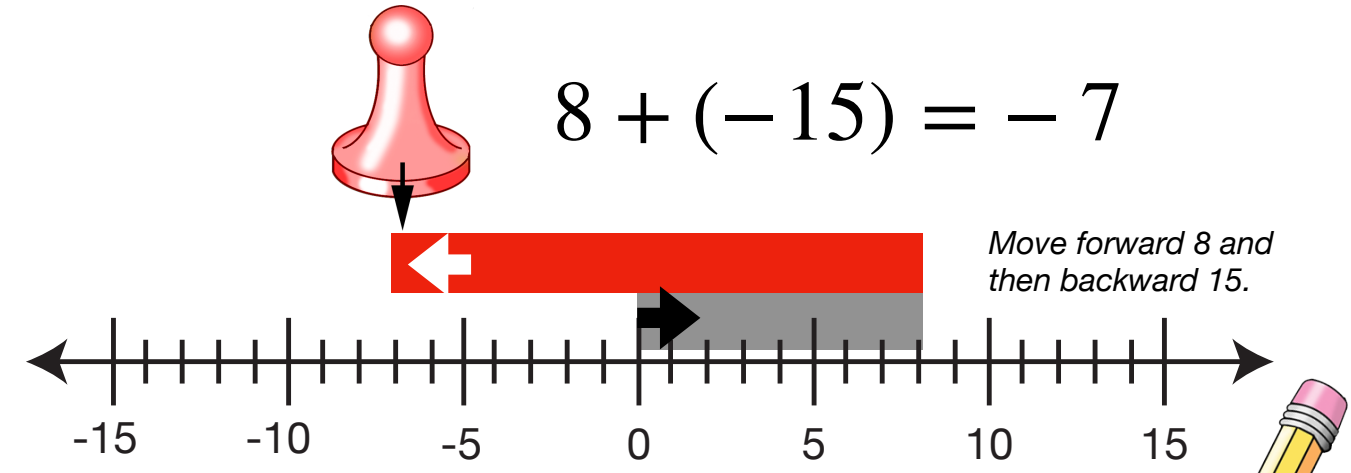
$$\begin{array}{r} 2 \\ -6 \\ \hline -4 \end{array}$$



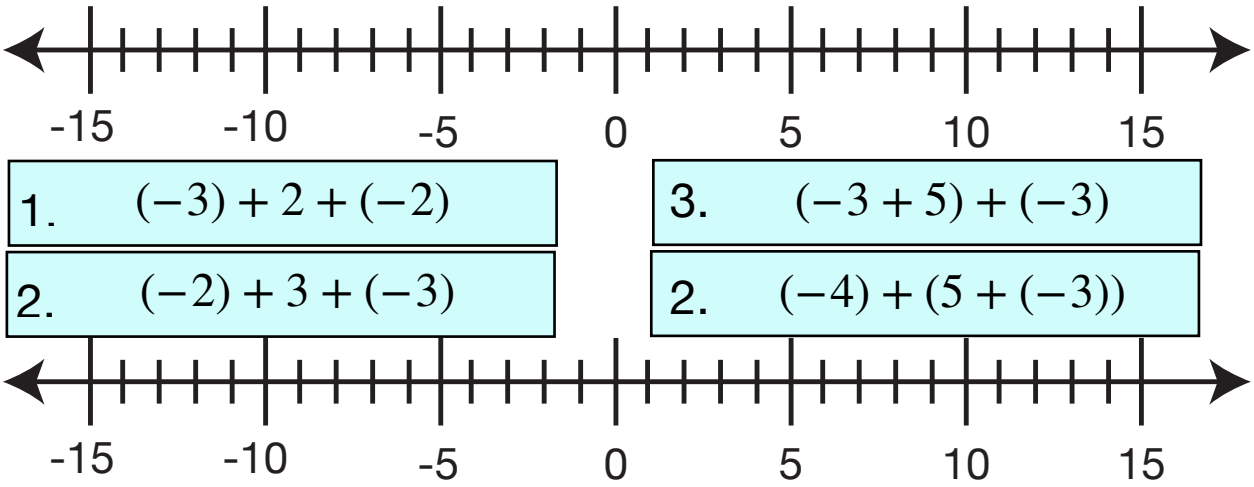
-2	-2	-6	-3	2	-2	-5	8	-9	-5
-6	-4	+7	6	-6	-6	-3	-6	3	5

Practice Adding Integers

One great way to visualize the addition of integers is to use a number line. Imagine moving the token on the number line. Here is an example:



1. $-3+5$
2. $-6+2$
3. $2+(-2)$
4. $(-4)+(-3)$
5. $(-5)+1$
6. $8+(-1)$
7. $(-3)+(-1)$
8. $(-3)+5$



1. $(-3)+2+(-2)$
2. $(-2)+3+(-3)$
3. $(-3+5)+(-3)$
2. $(-4)+(5+(-3))$

Expressing Binomials with Algebra Tiles

A binomial is an algebraic expression with two terms

$3x^2 + 4x$ $7x + 4$ $3x^2 + 3x$

$2x + 1$ $x^2 - 1$

Use the Algebra Tiles Playground to represent these binomials.

$2x^2 + 4x$ $4x^2 - 3x$ $-4x + 4$

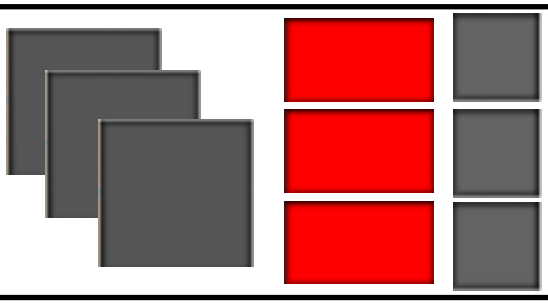
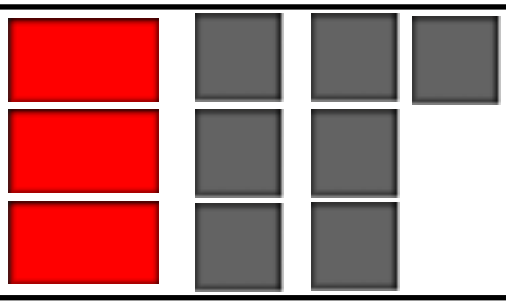
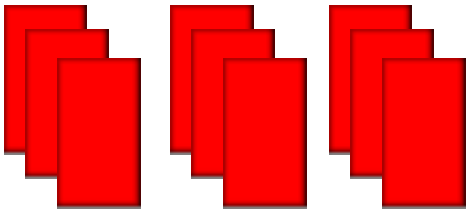
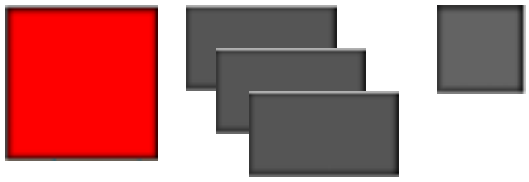
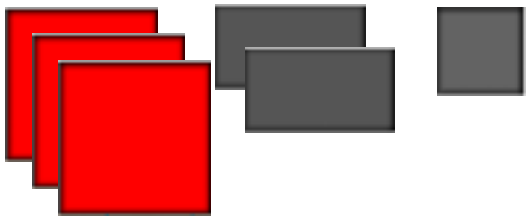
$-2x + 4$ $-x^2 - 2x$ $-3x^2 - 3x$

$-x^2 - x$ $-x^2 - 3x$ $-4x^2 + 3x$

$-2x^2 + 3x$ $-x^2 + 3x$ $-3x^2 + x$

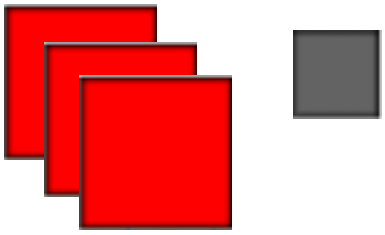
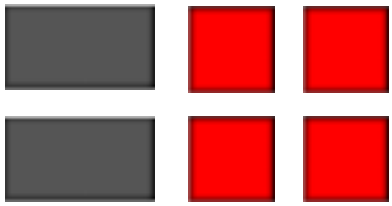
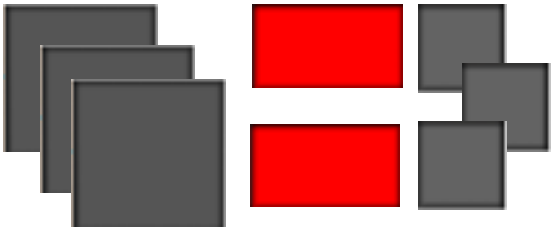
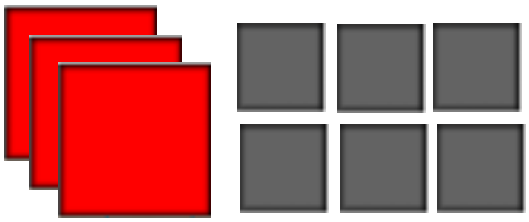
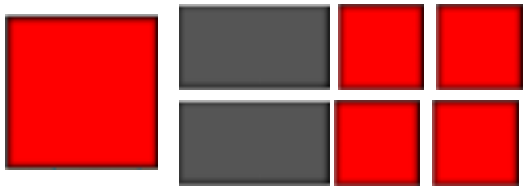
Evaluating Groups of Algebra Tiles

Any group of Algebra Tiles represents either a monomial, binomial or trinomial. Study the example then for each set shown decide if it is a monomial, binomial or trinomial. Write an expression for each set of tiles.

	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression: $3x^2 - 3x + 3$
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:

Evaluating Groups of Algebra Tiles

These groups of algebra tiles represent monomials, binomials or trinomials. For each problem, examine the set of tiles shown and decide if a monomial, binomial or trinomial is repressed. Write an algebraic expression for each set.

	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:
	Check one: ❑ monomial ❑ binomial ❑ trinomial	Expression:

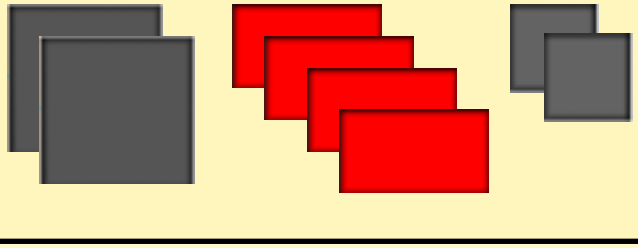
Addition of Polynomials

You can add polynomials by combining two groups of algebra tiles. For this activity use the Algebra Tiles Playground to find the sum of two polynomials. Write each of the polynomials represented in the smaller boxes and write the sum in the larger box.

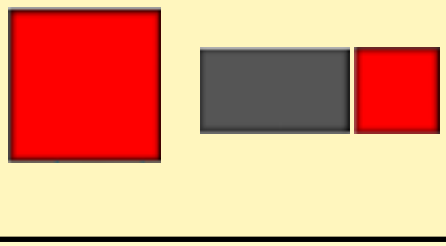
$$\begin{array}{r} 2x^2 - 4x + 2 \\ x^2 + x - 1 \\ \hline x^2 - 3x + 1 \end{array}$$

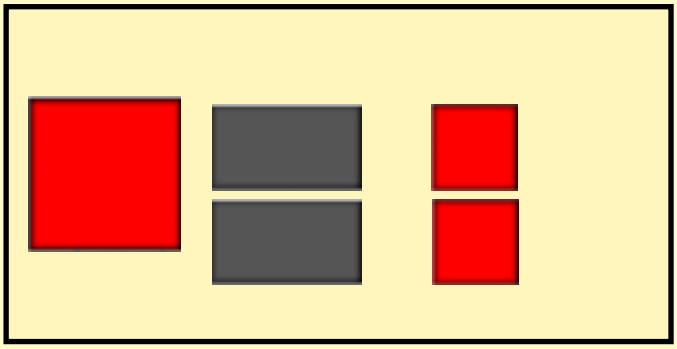
$x^2 - 3x + 1$

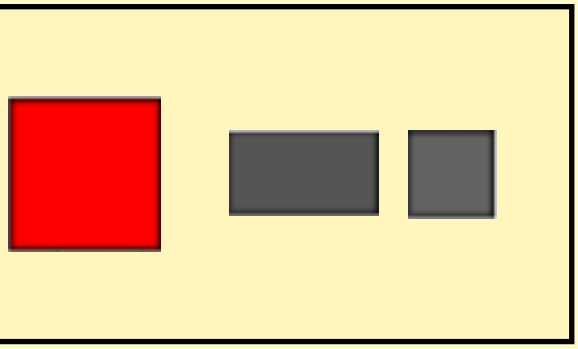
$2x^2 - 4x + 2$

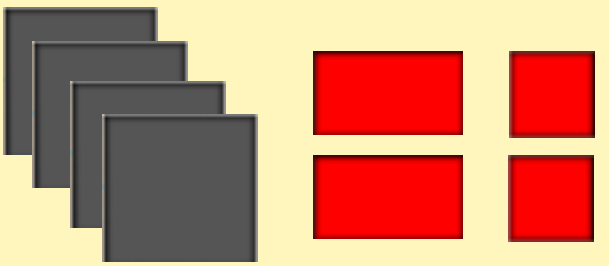


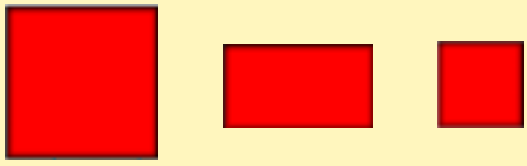
$x^2 + x - 1$



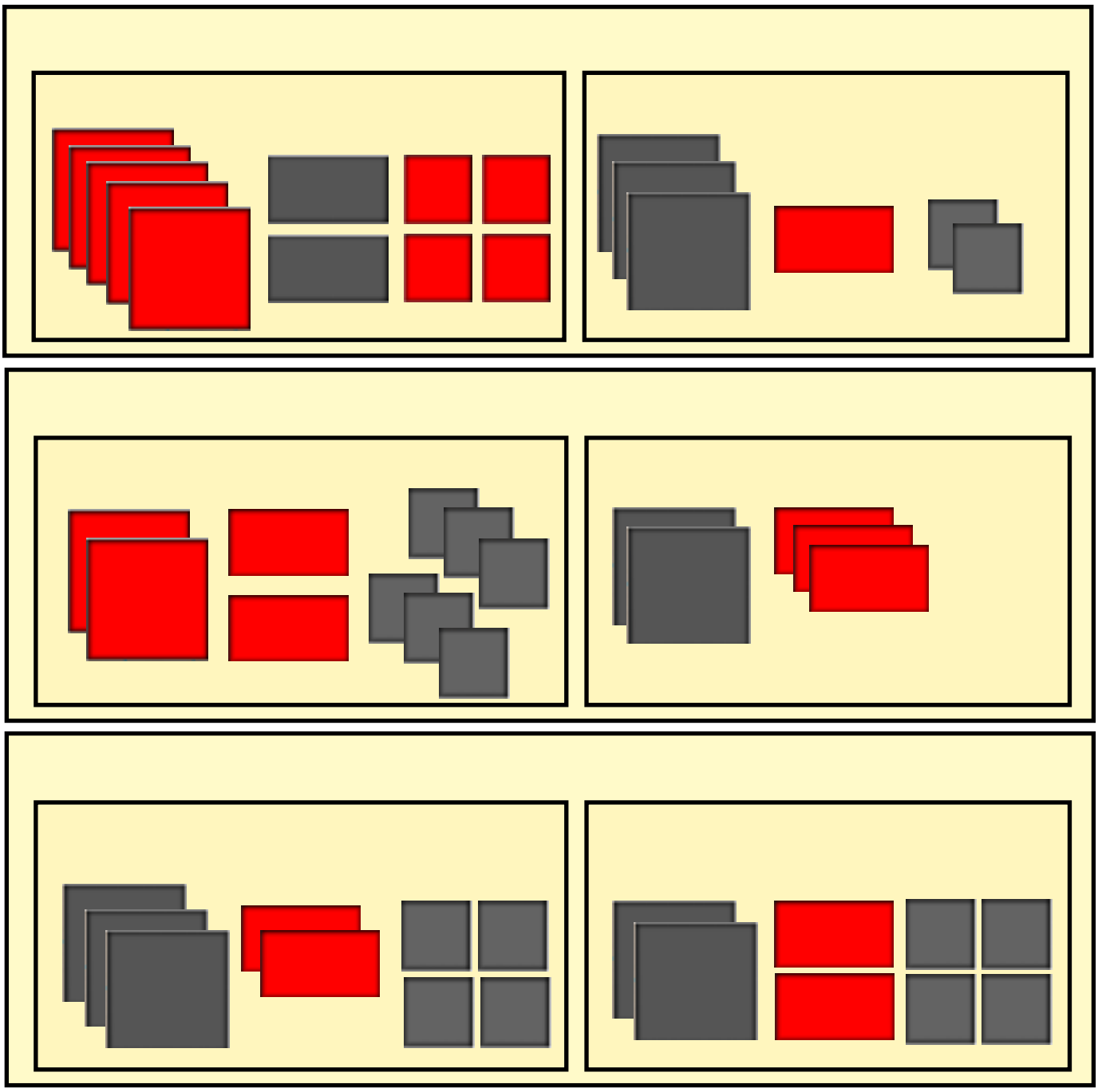








Write each polynomials represented in the smaller boxes and write the sum in the larger box.



Use the Algebra Tiles Playground to find the sum of these polynomials.

$3x^2 + 3x - 1$
$2x^2 + 5x - 4$

$5x^2 - 2x + 3$
$3x^2 - x - 3$

$2x^2 + 4x + 2$
$4x^2 + 5x + 6$

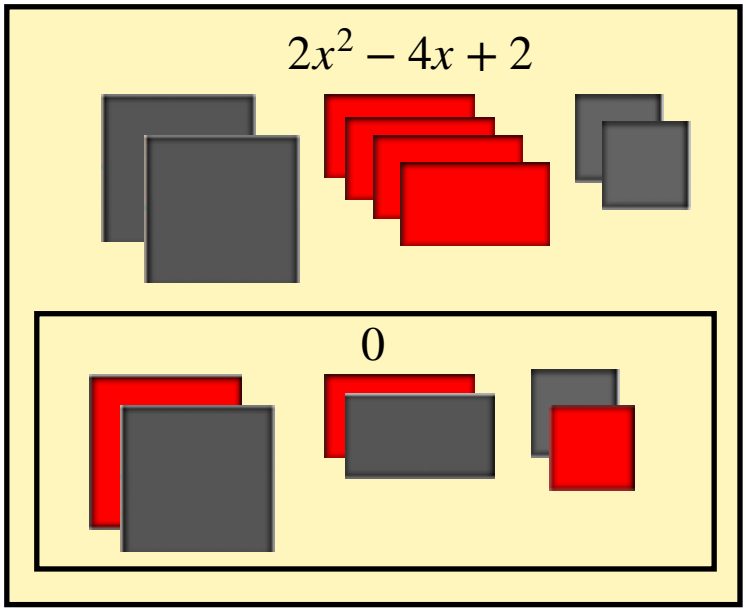
$x^2 + 2x + 5$
$3x^2 + x + 1$

Subtraction of Polynomials

The Algebra Tiles Playground can be used to study the process of subtracting polynomials.

Use Algebra Tiles to find the difference between two polynomials.

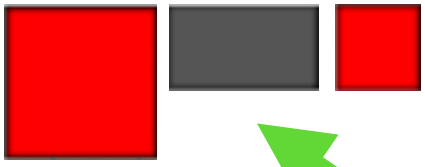
$$\begin{array}{r} 2x^2 - 4x + 2 \\ -x^2 + x - 1 \\ \hline 3x^2 - 5x + 3 \end{array}$$

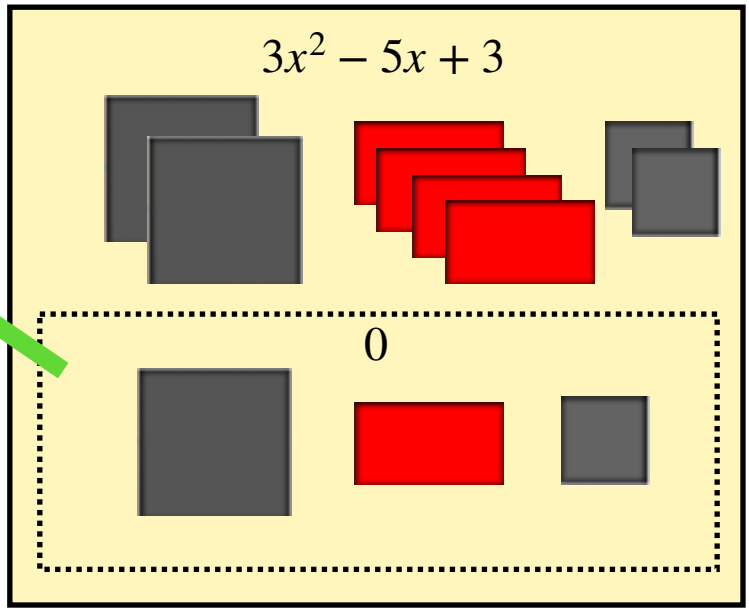


Add a set of tiles equal to zero (0) so that it is possible to subtract. Remember that the axiom of opposites states that a positive and a negative equals zero (0) if the absolute values of the two numbers are the same.

In order to subtract these polynomials a zero (0) must be added to the set.

$-x^2 + x - 1$



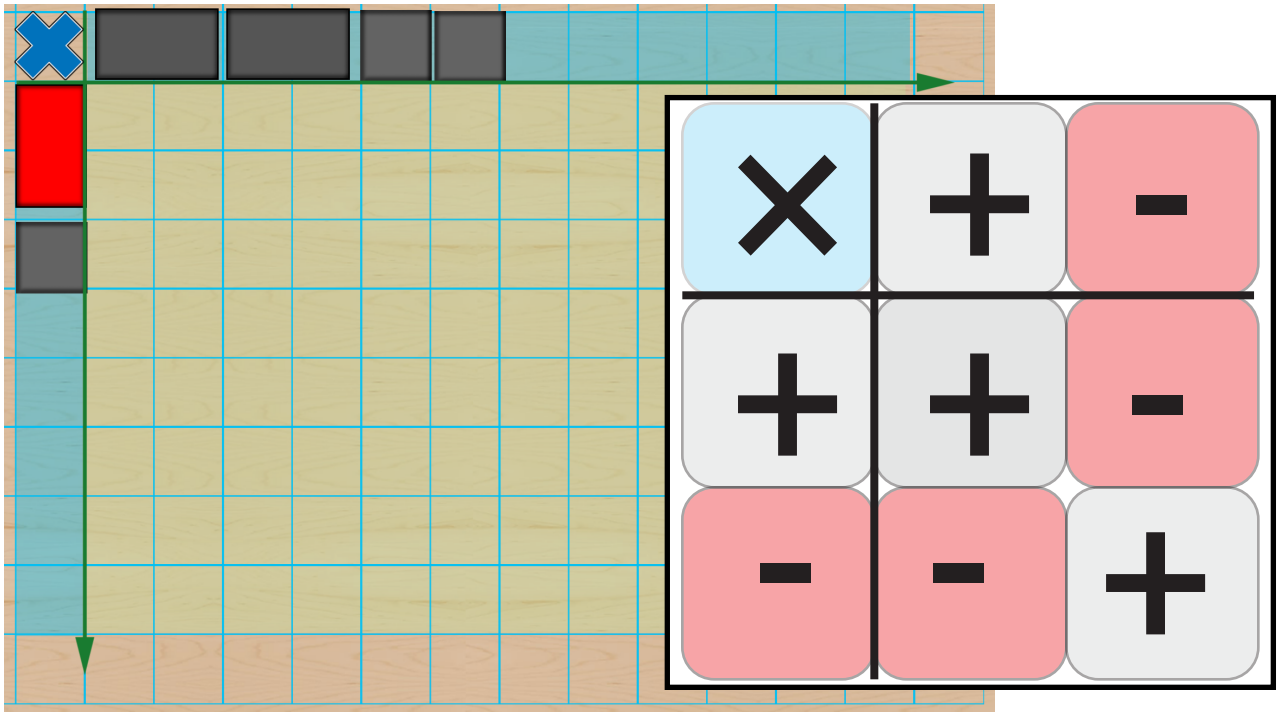


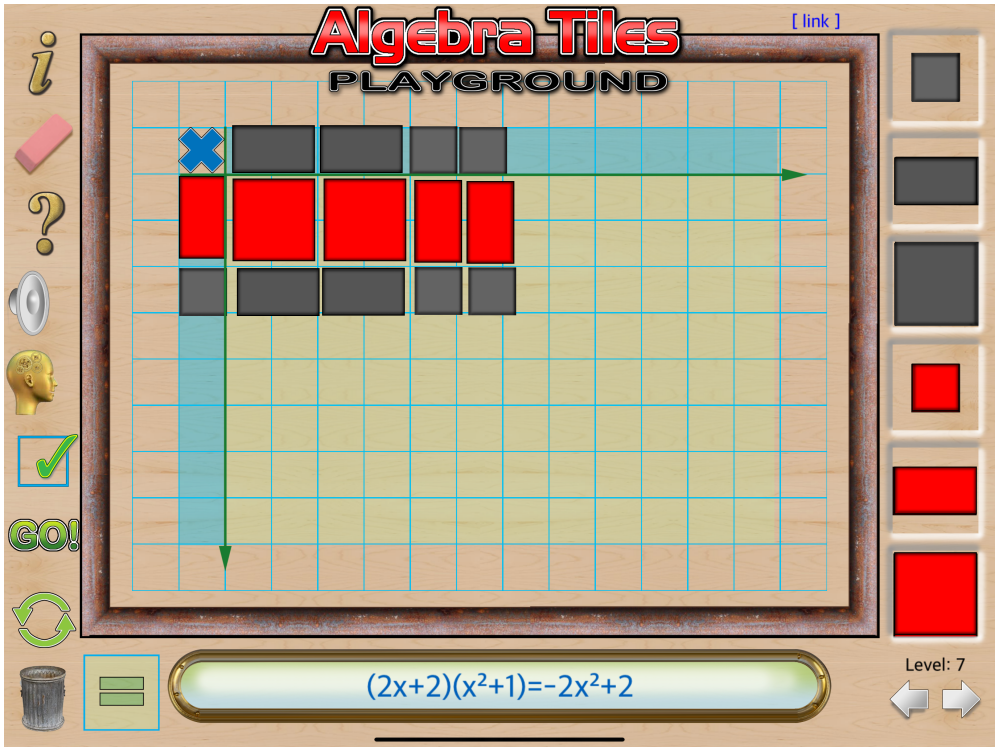
Once an appropriate set of tiles representing zero (0) has been added to the playground, the polynomial (subtrahend) can be removed from the Algebra Tiles Playground by sliding tiles off the grid.

Multiplication of Binomials

The Algebra Tiles Playground can be used to study how to multiply binomials and polynomials. Here's an example of how to set up a problem.

$$(2x + 2)(-x + 1)$$





Rules for Multiplication

- A positive integer times a positive integer yields a positive integer.
- A negative integers times a negative integer yields a positive integer.
- A positive integer times a negative integer yields a negative integer.

Multiplication of Binomials, Part 2

Use the Algebra Tiles Playground to solve this problem.

$$(3x + 1)(-2)$$

Practice

Solve these multiplication problems using the Algebra Tiles Playground.

$(-2x + 3)(x - 1)$

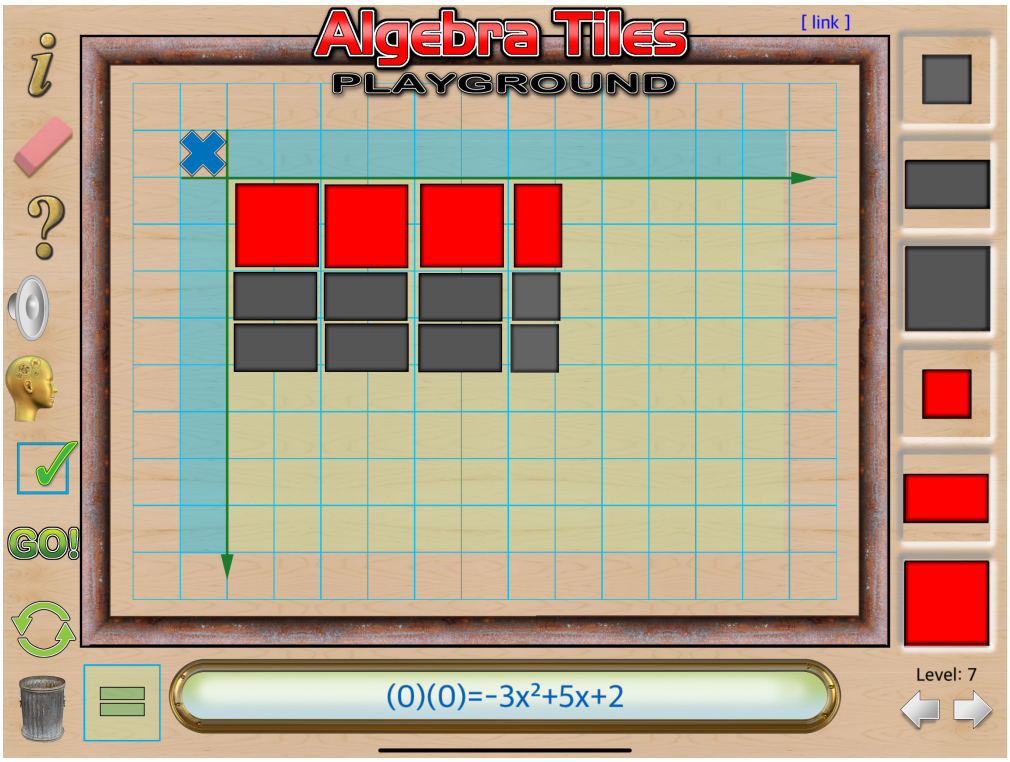
$(-x + 1)(x - 1)$

$(-2x - 1)(2x - 3)$

$(-x + 5)(x - 2)$

Factoring Polynomials

Using the Algebra Tiles Playground to factor a polynomial is similar to multiplying.



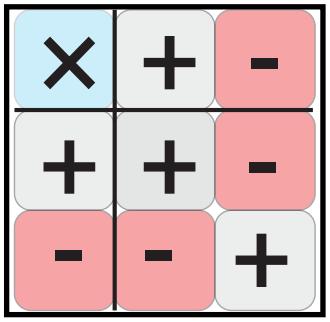
Let's factor this polynomial.

$$3x^2 + 5x + 2$$

The first step is to find a way to build the expression as a rectangle.

Apply the rules for multiplication of positive and negative integers to find the factors.

Use the rotate tool if necessary.



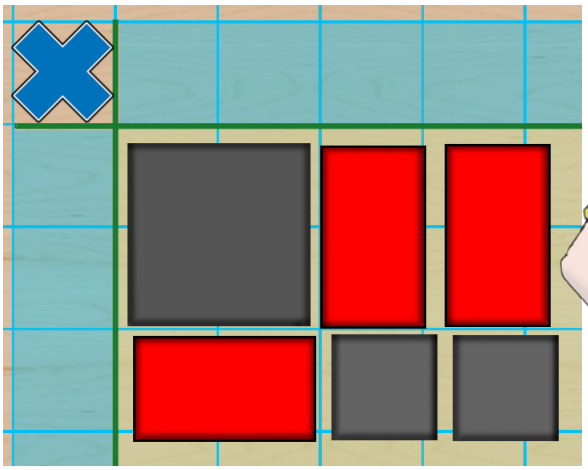
Practice Factoring Polynomials

Using the Algebra Tiles Playground to factor a polynomial is similar to multiplying.

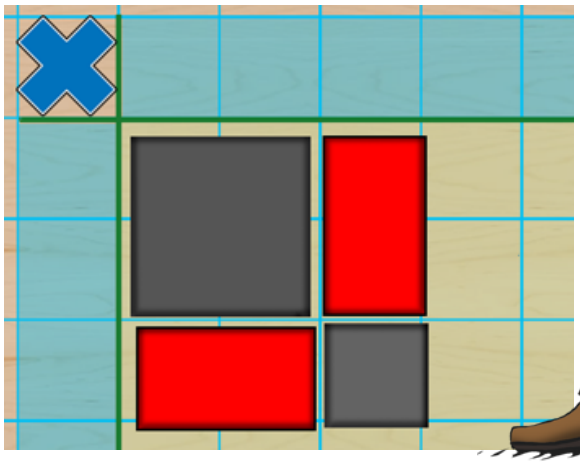
Polynomials	Factors	
$x^2 - 2x - 3$		
$x^2 + 2x$		
$6x^2 - 5x + 1$		
$2x^2 - 3x + 1$		
$6x^2 - 3x$		
$8x^2 + 2x$		

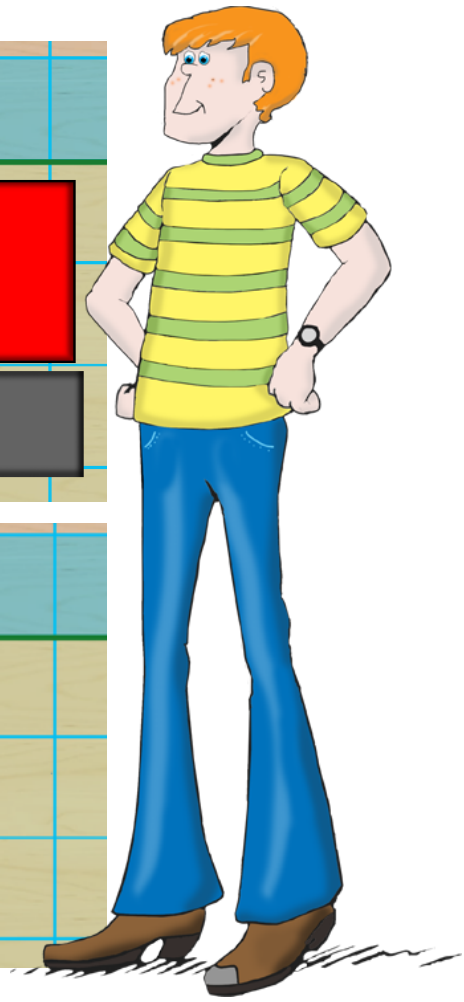
Find the factors of this polynomial.

$$x^2 - 3x + 2$$



$$x^2 - 2x + 1$$





Practice Problems: Operations with Integers

Draw a line to connect each problem with the correct solution.

$-10+4$	○	○	3
$8+(-5)$	○	○	1
$1+(-2)$	○	○	-5
$7+(-6)$	○	○	-6
$4-9$	○	○	-7
$2+(-9)$	○	○	-1
$(-10)+6$	○	○	-13
$(-4)+(-9)$	○	○	8
$(4)+(-9)$	○	○	-5
$(-1)-(-9)$	○	○	-4

Table Mania

Add across and down to get the magic number to put on the star. If the sum of the column and the row are different then an error was made.

4	−8	−4
−3	6	3
1	−2	−1

−3	6	
4	−10	

−3	4	
9	−8	

−3	2	
4	−5	

−8	5	
2	−6	

−1	4	
4	−8	